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研究課題名（英文） Analyzing the limits of computation using large scale linear programming

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研究成果の概要（和文）：本計画研究B02班のミッションは,PvsNP問題に対する数理計画的手法の追求であった.その中でも線形計画法(LP)は最も強力な手法であり非常に多くの最適化問題を実用的な計算時間で解くことができ,それゆえLPとLP定式化の複雑さの探求はPvsNP問題に対する有望なアプローチとなる.このような目標に対して我々は主に拡張定式化の複雑さ，および，通信計算量の２つの観点から研究を行った.3名の国内研究者と6名の海外研究者で編成されたチームによって，合計7つの具体的課題に取り組み成果を得た．これらの結果を，合計20編の論文うち16編は査読有,3編は投稿済査読中,および，8編の国際会議論文により発表した.

研究成果の概要（英文）：The mission of the B02 group was to use mathematical programming based methods to attack the P vs NP problem. The most powerful tool in mathematical programming is linear programming (LP), with which truly enormous problems can be solved to optimality in a reasonable amount of time.Therefore, studying the complexity of LP and LP formulations is a powerful tool in the study of the P vs NP problem. The two main tools we employed in our attack on this problem are extension complexity and communication complexity.With a team of 3 domestic and 6 international researchers we did research on 7 areas withing this field. This resulted in 20 papers of which 16 appear in refereed journals and 3 are under review.We also produced 8 papers which appear in conference proceedings.

研究分野：　最適化

キーワード：　幾何計算　最適化　数理計画法への応用

１．研究開始当初の背景

The mission of the B02 group of the ELC project was to use mathematical programming based methods to attack the P vs NP problem. The most powerful tool in mathematical programming is linear programming (LP), with which truly enormous problems can be solved to optimality in a reasonable amount of time. On the one hand, any problem in NP can be formulated as an LP. On the other hand, LP is P-complete which means that any problem in P can be solved by a polynomial size linear program. Therefore, studying the complexity of LP and LP formulations is a powerful tool in the study of the P vs NP problem. The two main tools we employ in our attack on this problem are *extension complexity* and *communication complexity*.

A vast array of important practical optimization problems can be modelled as high dimensional geometric problems. By high dimensional problems, we mean those where there is no fixed limit on the number of variables involved. Compared to low, fixed dimensional problems, high dimensional problems have the greatest potential for applications in many areas of engineering and science. Hence there has long been a great effort to design and implement software for this kind of problem. Most of this software was designed for the single core machines prevalent until relatively recently. However, parallel hardware is now ubiquitous so an important aspect of our project was to develop new methods to make efficient use of this powerful parallel processing power.

２．研究の目的

In order to concretize these goals, the B02 group was formed with a domestic and international group of experts with expertise in the required fields. The initial core domestic group consisted of Avis and Ueno from Kyoto University and Amano from Gunma University. The international group consisted of S. Fiorini (ULB) and H. Tiwary (Charles) for extension complexity, O. Friedmann (Munich) for LP complexity, and W. Cook (Waterloo), A. Deza (McMaster) and K. Fukuda (ETH) for polyhedra and integer programming.

A seminal paper of Yannakakis in 1991 combined the fields of extension complexity and communication complexity to obtain lower bounds on certain approaches to solving NP-hard problems by linear programming. The importance of this paper became clear in the early 2010s in a prize-winning STOC paper by Fiorini et al. The **first goal** of the B02 group was to apply and extend the results of these two papers.

An important research topic on convex polyhedra relates to the complexity of linear programming pivoting methods. In the early 2010s, Friedmann et al. brought powerful new tools from game theory and Markov decision processes to bear, and gave subexponential lower bounds for certain well established pivoting rules. The **second goal** of the B02 group was to extend these lower bounds to other history based pivot rules, to determine if matching upper bounds exist. We also planned to study the generalization of this approach to more abstract settings that go beyond polyhedral methods, including the computation of Nash equilibria and other linear complementarity problems.

An important feature of our work is that we fully incorporate creative theoretical insights into innovative highly parallelized and easy to use software for wide distribution. Although polyhedral methods have become an important tool in many areas of research in engineering and science, there was relatively little readily available software for the general researcher that exploits massive parallel architecture. Our **third goal** was therefore to greatly expand the tools available and computational power available to the general researcher.

３．研究の方法

The research consisted of 7 subprojects in major research areas that spanned the 5 year grant period: extension complexity, complexity lower bounds, lower bounds on history based pivot rules; computer analysis of Boolean functions; large scale parallel processing for geometric problems; ground metric learning; and directed cuts and open pit mining. We made good progress in each of these areas as we describe the next section. This progress was due in no small part to collaborative research with other teams in the ELC project as well as additional foreign based researchers. These latter included S. Langerman (ULB), D. Bremner (UNB), D. Rappaport (Queens), and L. Devroye (McGill) all of who visited Japan to further our research goals.

４．研究成果

We summarize below some of the major

results obtained in the seven subprojects described in Section 3.

(1) Extension Complexity

One of the exciting outcomes of the ELC grant was the introduction to Japan of the study of extension complexity. This subject is concerned with showing the limitations of linear programming in solving NP-hard problems. In June 2013, we organized an international workshop on the subject at Kyoto University. The workshop and resulting collaborations were made possible by ELC funds. These funds also supported subsequent annual trips to Kyoto University by Tiwary and has resulted in one conference paper and three papers accepted for publication in leading refereed journals in the field. A further paper has been submitted for review.

In their first collaboration, Avis and Tiwary extended earlier results of Fiorini et al. on the extension complexity of the cut polytope and related polyhedra. They first described a lifting argument to show exponential extension complexity for a number of NP-complete problems including subset-sum and three dimensional matching. They then obtained a relationship between the extension complexity of the cut polytope of a graph and that of its graph minors. Using this they were able to show exponential extension complexity for the cut polytope of a large number of graphs, including those used in quantum information and suspensions of cubic planar graphs. This work was initially presented at the prestigious ICALP conference [Conf. 2]. They extended these results resulting in a journal version [5] that appeared in Mathematical Programming.

In their second paper [3] they proposed a generalization of the extension complexity of a polyhedron Q. On the one hand, it is general enough so that all problems in P can be formulated as linear programs with polynomial size extension complexity. On the other hand, it still allows non-polynomial lower bounds to be proved for NP-hard problems independently of whether or not P=NP. The generalization, called H-free extension complexity, allows for a set of valid inequalities H to be excluded in computing the extension complexity of Q. They gave results on the H-free extension complexity of hard matching problems (when H are the odd set inequalities) and the traveling salesman problem (when H are the subtour elimination constraints).

In their third paper [1] they applied the framework they described in [3] to the traveling salesman problem (TSP). It was previously known that the extension complexity of the TSP polytope for the complete graph K_n is exponential in n even if the subtour inequalities are excluded. In this paper they studied the polytopes formed by removing other subsets H of facet defining inequalities of the TSP polytope. In particular, they considered the case when H is either the set of blossom inequalities or the simple comb inequalities. These inequalities are routinely used in cutting plane algorithms for the TSP. They showed that the extension complexity remains exponential even if one excludes these inequalities. In addition, they showed that the extension complexity of the polytope formed by all comb inequalities is exponential. In the proofs they introduced a subclass of comb inequalities, called (s,t)-uniform inequalities, which seem to be of independent interest. In their fourth and most recent paper they give a simple explicit compact formulation for the 2-SAT problem. This, in spite of the fact that the natural polytope for this problem has exponential extension complexity as they showed in [5].

Finally, Avis and Tiwary collaborated with D. Bremner and O. Watanabe to build a compiler that can convert any algorithm written in a basic pseudocode into a linear program. The initial goal here was to find a polynomial size polytope for the matching problem. Rothvoss had previously proved that no such polytope can be an extension of the Edmond’s polytope, so their work leads to completely different families of polytopes. The associated paper [10] has been submitted for journal publication. On the practical side, Avis and Bremner have implemented and distributed a prototype of the compiler. This open source software package is called *Sparktope*.

(2) Complexity Lower Bounds by Graph Partition

Amano and M. Shigeta investigated the fruitful connection between some graph theoretic problems and the communication complexity. They revealed in this work that constructing a good partition of a certain graph is essentially equivalent to getting a good lower bound on the amount of communication needed in a two-player game on graphs. Then by giving an explicit partition for the clique, they succeeded to improve the lower bound on the communication complexity of a graph problem introduced by Yanakkakis in the 90's. These results are reported in two journal papers [15,17].

(3) Study of Zadeh's pivot rules on hypercubes.

Klee and Minty showed that Dantzig's pivot rule for the simplex method requires exponential time in the worst case. This analysis was extended to most other known pivot rules. However, a family of history based pivot rules, introduced by Zadeh had defied analysis until Friedmann's breakthrough in 2010 when he showed a sub-exponential lower bound for the least entered rule. Avis and Friedmann obtained the first exponential lower bound for USOs and LPs that use Cunningham's history based rule, resulting in the paper [11] that appeared in Math. Programming.

(4) Computer Analysis of Boolean Functions

The investigation of the relationship between various complexity measures of Boolean functions is important to understand the nature of computation. The sensitivity is one of the most well-studied measures which intuitively represents the smoothness of a Boolean function.

Amano succeeded to enumerate all Boolean functions of small sensitivity with the aid of computers and revealed several interesting properties through the inspection of the results. He will present this at the international conference ISIT '17 [Conf 3].

(5) Multicore version of the lrs vertex enumeration code.

In 2012 Avis and Roumanis released the program *plrs* that is a shared memory parallelization of *lrs* and tested it with hardware using up to 64 processors at Avis' Geometric Computation Lab (GCL) at Kyoto University [Conf 1]. Over the ELC grant period this cluster grew to a total of 312 cores. The code *plrs* gave very satisfactory speedups up to about 16 cores but speedups were limited as additional cores were added. Furthermore, it was not able to run on the multi-machine cluster at GCL. From 2013 Charles Jordan (ELC, Hokkaido) and Avis started a new parallel implementation, *mplrs*, of *lrs* based on Message Passing Interface (MPI) that runs on clusters of machines. They experimented with various techniques to improve on the load balancing problem encountered with *plrs* for medium to large scale systems. Finally, they discovered a very simple technique they call budgeting that gave very good speedups over the entire cluster. This work is reported in [9], [12].

(6) Ground Metric Learning

Transportation distances have been used for more than a decade now in machine learning to compare histograms of features. They have one parameter: the ground metric, which can be any metric between the features themselves. Avis and M. Cuturi (ELC, Kyoto) considered algorithms that can learn the ground metric using only a training set of labelled histograms. They wrote the paper [7] on this that appeared in the Journal of Machine Learning.

(7) Directed cuts and open pit mining

This application area project involved the geometric computational approach to solving practical open pit mining problems. To minimize mine costs, models have been made that involve finding directed cuts in a network. Avis and C. Meagher (McGill) studied the related directed cut polyhedra and found classes of facets of these polyhedra that yield strong cutting planes. They presented this work at several international conferences and wrote a paper [2] that appeared in the Journal of Combinatorial Optimization.

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〔その他〕
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