様　式　C－19、F－19－1、Z－19　(共通)

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研究課題名（和文）幾何計算のための大規模並列化と数理計画法への応用

研究課題名（英文）Large scale parallelization for geometric computation and mathematical optimization

研究代表者
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研究成果の概要（和文）：1970年代以降のアルゴリズム研究のほとんどは、シングルプロセッサマシンで行われており、その結果、膨大な量の洗練された堅牢なソフトウェアが作成されました。残念ながら、このソフトウェアのほとんどは、今日人気のあるマルチプロセッサハードウェアで実行した場合、ほとんどメリットがありません。 このすべてのソフトウェアの再設計と再実装は、非常に時間とコストのかかる作業になる可能性があります。 この調査の目的は、既存の実装に本質的な変更を加えることなく,アルゴリズムを効率的に並列化することでした。これは既存のコードにラッパーとして並列化を追加することで実現されました。国際的な研究協力は主要な部分です。

研究成果の学術的意義や社会的意義
この期間中に、合計12の査読付き国際ジャーナル論文が発表されました。木検索方法の効率的な並列化を可能にするmtsパッケージを作成して配布しました。 lrslib多面体計算パッケージの範囲が大幅に拡張され、ほとんどの関数の並列化が含まれるようになりました。 lrslibは現在DebianLinuxの標準部分であり、研究者によって広く使用されています。特に、mplrsは現在、ほとんどの大規模な入力で利用できる最速の頂点列挙/凸包並列プログラムです。私たちのプログラムはすべてオープンソースです。

研究成果の概要（英文）：Most research in algorithms since the 1970s has been for single processor machines, resulting in a vast amount of sophisticated and robust software. Unfortunately most of this software profits very little when run on the now prevalent multiprocessor hardware. The redesign and re-implementation for all of this software will be an extremely time consuming and expensive task. The purpose of this research is to efficiently parallelize algorithms without any essential change to their existing implementation. This is achieved by adding parallelization as a wrapper to the existing code. This technique will be applied in particular to certain tree search algorithms used in optimization, artificial intelligence and machine learning. International research collaboration is a major part of our project.

研究分野：情報科学

キーワード：幾何計算　大規模並列処理　数理計画法への応用

様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９（共通）

１．研究開始当初の背景

A vast array of important practical optimization problems can be modelled as high dimensional geometric problems. By high dimensional problems, we mean those where there is no fixed limit on the number of variables involved. Compared to low, fixed dimensional problems, high dimensional problems have the greatest potential for applications in many areas of engineering and science. Hence there is tremendous importance in designing and implementing software for this kind of problem. Due to the complexity of high dimensional geometric problems, we mainly focus on the most structured class of objects, convex bodies, and polyhedra. Even here the inherent complexity is enormous, and some basic computational problems regarding convex polyhedra are still unanswered. Important recent theoretical breakthroughs in parallel processing have enabled useful software to be developed that has wide application. This research required further development and implementation in readily available easy to use software. That was our goal in this project.

２．研究の目的

I established the Geometric Computation Lab (GCL) at Kyoto University in 2010 using startup funds from JSPS to purchase the equipment for our parallel processing research and software development. During the course of this grant we planned to expand this lab to a distributed processing environment with roughly 500 cores. An important feature of our work is that we fully incorporate creative theoretical insights into innovative highly parallelized and easy to use software for wide distribution. Although polyhedral methods have become an important tool in many areas of research in engineering and science, there was relatively little readily available software for the general researcher that exploits massive parallel architecture. Our main goal was therefore to greatly expand the tools available and computational power available to the general researcher. To this end we established a three-pronged approach:

(1) Theoretical foundations. The effectiveness of parallel tree search clearly depends on the tree shape: a path will allow no parallelization at all. The tree shape depends on the problem to be solved. VE problems have very unbalanced trees, but similar reverse search algorithms such as triangulation generation give rise to balanced trees. With Luc Devroye (Mcgill), a world expert on random trees, we planned to analyze the behavior of the budgeting method developed by Charles Jordan and myself. In particular, we planned to determine the expected number of subtrees generated (as these cause overhead) and the probability that all cores are working at a given time. We planned to do this analysis for different random tree models, corresponding to the types of trees met in practice.

(2) Geometric computation. Reverse search is one of the fundamental methods used in geometric computation. Besides the vertex enumeration problem, it can be used for such diverse problems as generating triangulations, cells in an arrangement, semi rigid structures, Euclidean spanning trees, etc. There is a large amount of code available for these problems that does not benefit from multicore hardware. With Jordan, our goal was to make a more generic form of the mplrs wrapper that allows the simple and effective parallelization of these existing codes.

(3) Discrete optimization. Such famous problems as the travelling salesman problem (TSP) and the max cut problem can be solved optimally only by massive tree search. With Cook, Imai and Tiwary we planned use massive parallelization to attack important unsolved problems, such as facet enumeration problems arising in quantum computing.

３．研究の方法

Our research plan involved 5 overlapping projects each involving myself and a subgroup of the team. These projects span theory, implementation, experimentation, application and distribution of results.

(1) Theoretical analysis of parallel tree search on a variety of random tree models (Devroye, Jordan)
Tree search seems ideally suited for parallel computation: one simply enumerates tree nodes at some fixed depth and places them in a list L. A scheduler assigns nodes from L to separate processors for evaluation of the corresponding subtrees. The major problem is load balancing when the search tree is unbalanced. Jordan and I had previously proposed budgeting as a method of solving this problem. This subproject concerned a study of the behavior of budgeting on random trees. The questions we attempt to solve are: (a) What is the expected number of nodes placed on L? (b) What is the proportion of the running time that L is empty. Simultaneously we planned to get empirical results for a variety of random tree models using modifications of our parallel software.

(2) Parallelization of existing tree search algorithms for geometry and optimization (Jordan)
The main goal was to write a general easy to use wrapper to parallelize existing single core codes.

(3) Application to problems in quantum information theory (Imai, Tiwary)
Quantum information theory is a source of very large hard to solve vertex enumeration and convex hull problems. Our goal is to work with established researchers in this area with a view to providing them with new parallel software to attack previously unsolved problems.

(4) Applications in discrete optimization (Bremner, Cook, Tiwary)
Extension complexity is the attempt to model hard combinatorial polyhedra as the projection of higher dimensional polyhedra that have short combinatorial descriptions. Although this is not possible with NP-hard problems, it is possible for certain problems that are polynomially solvable, such as the matching problem. We planned to write a compiler which would create polynomially sized extended formulations for any problem that could be solved efficiently.

(5) Laboratory for Parallel Geometric Computation (LPGC) (all members)
The laboratory was planned to be extended to 512 cores by the end of FY2021. All four of the above projects make extensive use of the LPGC.

４．研究成果

The results obtained for each of the 5 projects is described briefly, where the publications referenced are listed below.

(1)In Avis-Devroye, *Algorithmica* (2020) we provided tight analysis of the budgeting method for balancing tree search on the family of Galton-Watson trees. We answered both questions (i)(a) and (b) above giving strong theoretical justification for the empirically observed efficiency of this method.

(2) In Avis-Jordan, *Mathematical Programming Computation* (2018) we described the recently released *mplrs* code which parallelized *lrs*, a reverse search code for vertex enumeration. With sufficiently many cores, for most inputs it was shown to outperform the other state of the art codes tested. In Avis-Jordan, *Optimization and Software Methods* (2019) we presented the *mts* software package which we had recently distributed. This package allows parallelization of existing tree search codes, subject to certain limitations, with only minimal changes to the original code. We gave examples from reverse search codes showing exceptional speedup using *mts*. In Jordan-Joswig-Katner (2017) these techniques were used to develop a parallel high-dimensional triangulation enumeration program.

(3) In joint work with Hernández-Cuenca, "On the foundations and extremal structure of the holographic entropy cone", arXiv:2102.07535 (2021), we studied the Holographic Entropy Cone arising from the study of quantum gravity and information. Using various tools developed in this project, including *mplrs*, we were able to completely compute this cone for 6 terminals directly from first principles. We also provided a review of earlier

work in the Physics community in a language better understood by the Discrete Optimization community to encourage future research. This work has been submitted for journal publication.

(4) In Avis-Tiwary, *Optimization Letters* (2017) and *European J. of Combinatorics* (2019), we presented new research on extension complexity. The first paper involved a strengthening of the definition to allow certain additional families of constraints to be included even if they had high complexity. The second paper gave an efficient extended formulation for 2SAT. In Avis-Bremner, *Optimizations and Software Methods* (2021), a compiler (*Sparktope*) was built to produce Linear Programs (LPs) from algorithms that were input in a simplified programming language. The series of LPs generated were polynomial in size whenever the input programs would terminate in a polynomial number of steps. As a non-trivial example, we implemented Edmonds' matching algorithm and converted into the first polynomially sized LPs to solve the matching problem. The *Sparktope* compiler is freely available.

(5) In 2015, at the beginning of this grant, the LPGC contained 3 machines, *mai*12,20,64, with a total of 92 cores. This was insufficient to test how well our budgeting technique would scale to larger systems. Accordingly, over the course of the grant, the additional machines *mai*32a,b,c,d,ef,*mai*24,*mai*48,*mi*64a,32a,64b were added bringing 380 additional cores and total cluster size of 472 cores. This relatively large cluster showed the extremely good performance of our codes and allowed us to solve various previously unsolved problems in geometric computation.

A major achievement of the grant was the extension of the *lrslib* package to include a number of other functions typically required in polyhedral computation. In June 2020 we released version 7.1 which included our first parallel implementation of redundancy removal, the extract option to remove linearities and a cross reference function between vertices/rays and facets. It was our first C only implementation containing the parallel code *mplrs*. This was followed in March 2022 by the release of version 7.2. This release included a parallel implementation of the Fourier-Motzkin method for obtaining the projection of a set of half-spaces. It also included functions for determining if an inequality is redundant in terms of computing the projection of half-spaces to lower dimensions. This is done without computing the projection itself by use of a SMT (Satisfiablity Modulo Theories) solver.

Since C does not do integer overflow checking there is a possibility of incorrect results being produced. However direct use of extended precision arithmetic is much slower than fixed precision. One of the features of *lrslib* is a hybrid arithmetic package that can be called to do exact integer arithmetic in a hybrid manner with full overflow checking. The computation begins using standard 64-bit precision and then switches to 128-bit and finally extended precision arithmetic (such as GNU GMP) as necessary. Since this arithmetic package can be useful for a wide variety of applications, we released the necessary functions in the small library *lrsarith*. The details were published in Avis-Jordan (2021).

International collaboration was a major part of the success of this project in achieving its goals. Making visits to Japan to further our research were David Bremner (UNB), William Cook (Waterloo), Luc Devroye (McGill) and Hans Raj Tiwary (Charles). These visits were regrettably curtailed during the last two years of the grant period due to the Covid-19 virus.

５．主な発表論文等

〔雑誌論文〕　計15件（うち査読付論文　12件／うち国際共著　15件／うちオープンアクセス　9件）

１．著者名 K. Yamanaka, D. Avis, T. Horiyama, Yoshio Okamoto, R. Uehara and T. Yamauchi	４．巻 online
２．論文標題 Algorithmic Enumeration of Surrounding Polygons	５．発行年 2020年
３．雑誌名 Discrete Applied Mathematics	６．最初と最後の頁 1-19
掲載論文のDOI（デジタルオブジェクト識別子） 10.1016/j.dam.2020.03.034	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 D. Avis and L. Devroye	４．巻 82
２．論文標題 An Analysis of Budgeted Parallel Search on Conditional Galton-Watson Trees	５．発行年 2020年
３．雑誌名 Algorithmica	６．最初と最後の頁 1329-1345
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s00453-019-00645	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 D. Avis and C. Jordan	４．巻 10
２．論文標題 mts: A light framework for parallelizing tree search codes	５．発行年 2018年
３．雑誌名 Optimization and Software Methods	６．最初と最後の頁 267-302
掲載論文のDOI（デジタルオブジェクト識別子） 10.1080/10556788.2019.1692344	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 D. Avis and H. R. Tiwary	４．巻 80
２．論文標題 Compact linear programs for 2SAT	５．発行年 2019年
３．雑誌名 European Journal of Combinatorics	６．最初と最後の頁 17-22
掲載論文のDOI（デジタルオブジェクト識別子） 10.1016/j.ejc.2018.02.011	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 D. Avis and C. Jordan	４．巻 10
２．論文標題 mplrs: A scalable parallel vertex/facet enumeration code	５．発行年 2018年
３．雑誌名 Mathematical Programming Computation	６．最初と最後の頁 267-302
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s12532-017-0129-y	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 D. Avis, D. Bremner, H. R. Tiwary and O. Watanabe	４．巻 online
２．論文標題 Polynomial size linear programs for problems in P	５．発行年 2019年
３．雑誌名 Discrete Applied Mathematics	６．最初と最後の頁 online
掲載論文のDOI（デジタルオブジェクト識別子） 10.1016/j.dam.2019.03.016	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 D. Avis and D. Bremner	４．巻 36
２．論文標題 Sparktope: linear programs from algorithms	５．発行年 2021年
３．雑誌名 Optimization and Software Methods	６．最初と最後の頁 279-300
掲載論文のDOI（デジタルオブジェクト識別子） 10.1080/10556788.2020.1864370	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 C. Jordan, M. Joswig and L. Kastner	４．巻 25
２．論文標題 Parallel Enumeration of Triangulations	５．発行年 2018年
３．雑誌名 The Electronic Journal of Combinatorics	６．最初と最後の頁 1-27
掲載論文のDOI（デジタルオブジェクト識別子） 10.37236/7318	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 D. Avis and S. Hernandez-Cuenca	4．巻 1
2．論文標題 On the foundations and extremal structure of the holographic entropy cone	5．発行年 2020年
3．雑誌名 arXiv:2102.07535	6．最初と最後の頁 1-32
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 無
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 D. Avis and H. R. Tiwary	4．巻 153
2．論文標題 On the Extension Complexity of Combinatorial Polytopes	5．発行年 2017年
3．雑誌名 Mathematical Programming B	6．最初と最後の頁 95-115
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/978-3-642-39206-1_6	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 D. Avis and C. Jordan	4．巻 1
2．論文標題 lrsarith: a small fixed/hybrid arithmetic C library	5．発行年 2021年
3．雑誌名 arXiv:2101.12425	6．最初と最後の頁 1-14
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 無
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 D. Avis and D. A. Hoang	4．巻 2022
2．論文標題 On Reconfiguration Graph of Independent Sets under Token Sliding	5．発行年 2017年
3．雑誌名 arXiv:2203.16861	6．最初と最後の頁 1-17
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 無
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 D. Avis and O. Friedmann	4．巻 161
2．論文標題 An Exponential Lower Bound for Cunningham's Rule	5．発行年 2017年
3．雑誌名 Mathematical Programming	6．最初と最後の頁 271-305
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s10107-016-1008-4	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 D. Avis and C. Meagher	4．巻 31
2．論文標題 On the Directed Cut Cone and Polytope	5．発行年 2016年
3．雑誌名 J. Combinatorial Optimization	6．最初と最後の頁 1685-17-8
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s10878-015-9863-0	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 C. Jordan and T. Zeugmann	4．巻 4
2．論文標題 The Kahr-Moore-Wang Class Contains Untestable Properties	5．発行年 2016年
3．雑誌名 Baltic Journal of Modern Computing	6．最初と最後の頁 736-752
掲載論文のDOI（デジタルオブジェクト識別子） 10.22364/bjmc.2016.4.4.11	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

〔学会発表〕　計1件（うち招待講演　1件／うち国際学会　1件）

1．発表者名 C. Jordan
2．発表標題 Parallel vertex and facet enumeration with mplrs
3．学会等名 Algebraic Statistics and Symbolic Computation（招待講演）（国際学会）
4．発表年 2017年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

lrs home page
http://cgm.cs.mcgill.ca/~avis/C/lrs.html

Sparktope home page
https://gitlab.com/sparktope/sparktope

mptopcom home page
https://polymake.org/doku.php/mptopcom

Parallel enumeration of triangulations
https://www.polymake.org/mptopcom

mts home page
http://cgm.cs.mcgill.ca/~avis/doc/tutorial.html

6．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考
研究分担者	ジョーダン　チャールズハロルド (Jordan Charles) (60647577)	小樽商科大学・情報科学研究院・助教 (10101)	

7．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計2件

国際研究集会	開催年
Workshop on Graphs and Geometric Algorithms	2019年～2019年
Workshop on Graphs and Geometric Algorithms	2018年～2018年

8．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関