様 式 C－19、F－19－1、Z－19 （共通）

科研費 KAKENHI

科学研究費助成事業　　研究成果報告書

令和　元 年　 ６ 月 １８ 日現在

機関番号：１０１０１
研究種目：基盤研究(C)（一般）
研究期間：2016～2018
課題番号：１６Ｋ０５０５１
研究課題名（和文）On Generalized Pure Braid Groups

研究課題名（英文）On Generalized Pure Braid Groups

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交付決定額（研究期間全体）：（直接経費）　　3,300,000円

研究成果の概要（和文）：配置MS(n,k)は、一般的な位置配置を形成することができないk空間内のn個の超平面の所与の配置の平行移動の集合として定義される。MS(n,k)の最初の位相不変量を調べたところ、Betti数の漸近的振る舞いとMS(n,k)の元を与える生成関数が見つかりました。
さらにMS(n,k)上のチェンバーの数が、Bruhatの次数B(n,k)の組の要素数の下限であることを証明し、MS(k,k+3)のフリーネスを調べました。
配置Aが十分に一般的であるならば、MS(n,k,A)の交差格子はAから独立している。そのような配置は非常に一般的な配置と呼ばれ、そして配置はMS(n,k)と表される。

研究成果の学術的意義や社会的意義
We studied the topological invariants of the Manin Schechtmann arrangements MS(n,k,A) in non very generic case. This provided a way to study the difficult problem of special configurations of points in the projective space, one of the main problems in phisics and geometry, using combinatorics.

研究成果の概要（英文）：Manin Schechtmann arrangements MS(n,k) are defined as the set of parallel translates of a given arrangement of n hyperplanes in a k-space which fail to form a general position arrangement. We studied the First topological invariants of MS(n,k), we found generating function that gives Asymptotic behaviour of the Betti numbers and chambers of MS(n,k). Moreover we proved that the number of chambers on MS(n,k) is a lower bound for the number of elements in the set of Bruhat orders B(n,k) and studied freeness of MS(k+3,k).If the arrangement A is generic enough then the intersection lattice of MS(n,k,A) is independent of A. Such arrangements are called very generic arrangements, and then the Manin-Schechtmann arrangements are noted MS(n,k). In this particular case, the intersection lattice of MS(n,k) have been explicitly described by Athanasiadis.

Using similar arguments as in a paper of Salvetti and Settepanella, we give description of betti numbers of MS(n,k).

研究分野：Combinatorics

キーワード：Discriminantal Arrang.　Bruhat Orders　Braid Groups

１．研究開始当初の背景

The topology of ordered and unordered configuration spaces of h distinct points in a manifold M is a very old subject. The simplest and best known example are configuration spaces of points in real plane studied by Artin more than 75 years ago and that lead to discovery of braid groups. There is a huge literature on these two groups (more than 68000 results only on google scholar) that are very famous and have been widely studied in mathematics and physics with an enormous range of applications. This project was aimed to study a very natural generalization of ordered and unordered configuration spaces, that is the topological structure of configuration spaces of linear subspaces in projective space. From the point of view of fundamental group the case in which linear subspaces have codimension 1 is the most interesting in the larger hierarchy of configuration spaces consisting of collections of linear subspaces having arbitrary codimension since if codimension is greater than one such configuration spaces are simply connected as proved by the P.I. in a couple of joint paper with S. Manfredini, co-researcher in this project. Those spaces are closely related to arrangements of hyperplanes introduced by Manin and Schechtman (see \cite{man}) and known as Discriminantal Arrangement.

２．研究の目的

The purpose of this research was to study the topological invariants of the Manin Schechtmann arrangements MS(n,k,A) and their relations with the higher Bruhat orders. These arrangements can be defined as the set of parallel translates of a given arrangement of n hyperplanes in a k-space which fail to form a general position arrangement.

３．研究の方法

The research has been carried out thanks to several meetings in Italy, Germany and Japan.

４．研究成果

We studied the First topological invariants of MS(n,k), we found generating function that gives Asymptotic behaviour of the Betti numbers and chambers of MS(n,k).

Moreover we proved that the number of chambers on MS(n,k) is a lower bound for the number of elements in the set of Bruhat orders B(n,k) and studied freeness of MS(k+3,k).

If the arrangement A is generic enough then the intersection lattice of MS(n,k,A) is independent of A. Such arrangements are called very generic arrangements, and then the Manin-Schechtmann arrangements are noted MS(n,k). In this particular case, the intersection lattice of MS(n,k) have been explicitly described by Athanasiadis.

Using similar arguments as in a paper of Salvetti and Settepanella, we give description of betti numbers of MS(n,k). Still mimicking the construction of Salvetti and Settepanella, we were able to give a recursive formula for the number of chambers in MS(n,k).

The asymptotic behaviour of the i-th Betti number of MS(n,k) when n grows has also been given. We also considered generating functions for the invariants of discriminantal arrangmenents of n hyperplanes in k-dimensional space for fixed k. The invariants studied are the number of chambers over reals, the Betti numbers of fixed codimension in the case of arrangements over complex. Aa a result one obtains rational functions which we were able to get explicitely in some cases and we are presently working on extending these cases to more general situations. An intersecting outcome is that these rational functions have resemblance to the generating function involving Ehrhart polynomial associated with the polytopes. We are looking for a direct explanation of this similarity. The results presented in this section will give rise to a paper entitled: Enumerative study of discriminantal arrangements.

Relation with the higher Bruhat orders. In the same paper , Manin and Schechtman also introduced Higher Bruhat order B(n,k) as the set of all the admissible up to equivalence total orders on C(n,k), the set of all subsets of k elements in the set {1, ..., n}. A total order on C(n,k) is called admissible if it induces either lexicographic or the inverse lexicographic order in each K-packet of the form $P(K)=\{J~\mid~J \in C(n,k), J \subset K \}, K \in C(n,k+1)$ and two admissible orders are equivalent if they differ by an interchange of two neighbours which do not belong to a common packet. In this same paper Manin and Schectman give a geometric interpretation of this combinatorial notion in terms of the Discriminantal arrangement. Essentially they argue that the Higher Bruhat orders model the set of minimal paths through Discriminatal arrangement. Subsequently Ziegler shows that we have to choose a cyclic arrangement for this and that in general the poset B(n,k) contains non-realizable elements which might not occur in the path space of the arrangement. This raises the question on the relation between the Discriminantal arrangements and Higher Bruhat orders. Later on Felsner and Ziegler proved that there is an inclusion of the set of vertices of the fiber polytope associated with the polytope projection of the n-dimensional cube onto the Zonotope Z=Z(V) dual to the configuration V of n vectors in k-dimensional real space into B(n,k). As a Corollary of this result and of two other Theorems due to Falk and Bayer and Brandt we proved that the number of chambers on MS(n,k) is a lower bound for the number of elements in B(n,k). The above results, combined with the Athanasiadis and Libgober and results, allowed a full understanding of the first non trivial case of Discriminantal arrangement, MS(k+3,k) including the study of its freeness. Moreover a generic 2-dimensional section in $C^{k+3,k}$ of the arrangement MS(k+3,k) is a line arrangement consisting of all diagonals of a polygon with k+3 vertices with the additional condition that, beside the vertices of the polygon, all other intersection points are double or triple points. Then using the above results on the combinatorics of MS(k+3,k) we provided a lower bound for the number of triple intersections in this line arrangement. The results presented in this section will give rise to a paper entitled: Discriminantal arrangments, Bruhat orders and line arrangements.

５．主な発表論文等

〔雑誌論文〕（計３件）

①S. Sawada, S. Settepanella and S. Yamagata, Pappus’s Theorem in Grassmannian GR(3;Cn), Ars Mathematica Contemporanea, 16 (2019) 257-276. Peer reviewed DOI なし

②S. Sawada, S. Settepanella and S. Yamagata, Discriminantal arrangement, 3 3 minors of Plücker matrix and hypersurfaces in Grassmannian Gr(3; n), Comptes Rendus Mathematique Volume 355, Issue 11(2017), 1111-1200. Peer reviewed DOI なし

③A. Libgober and S. Settepanella, Strata of discriminantal arrangements, *Journal of Singularities* Volume in honor of E. Brieskorn 18 (2018) 441-456 Peer reviewed DOI なし

〔学会発表〕（計　３　件）

〔図書〕（計　０　件）

〔産業財産権〕

○出願状況（計　0　件）

名称：
発明者：
権利者：
種類：
番号：
出願年：
国内外の別：

○取得状況（計　0　件）

名称：
発明者：
権利者：
種類：
番号：
取得年：
国内外の別：

〔その他〕

ホームページ等

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