3版

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科学研究費助成事業　　研究成果報告書

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研究課題名（和文）Photonic Topological Insulators: Dynamical and Topological Effects

研究課題名（英文）Photonic Topological Insulators: Dynamical and Topological Effects

研究代表者
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交付決定額（研究期間全体）：（直接経費）　2,700,000円

研究成果の概要（和文）：本研究過程で8つの査読済みトップジャーナル論文、分担著書籍と書籍を各1冊出版、他に査読修正中論文有り。チリ・日・韓・蘭の6人の共同研究者を含む。トポロジカル絶縁体に係るワークショップを三度主催した。Project-1と3では非常に良い進展があった。Project-1では従来の結果を修正し電磁波の実数値の制約を適切に考慮する必要があった。Project-3での予想外の結果は金属-誘電体界面での表面波のトポロジー的性質；エルミート類似体を持たず、これまで知られていないバルク境界対応を特徴としたものであった。トポロジーと非線形効果（Project-2）間の相互作用は現時点で結論付けられていない。

研究成果の学術的意義や社会的意義
The two biggest contributions were: (1) The real-valuedness of classical waves leads to a constraint which needs to be distinguished from symmetries in the classification. (2) The discovery of a new type of bulk-boundary correspondence with no quantum analog has opened up a new avenues of research.

研究成果の概要（英文）：In the course of the project, I have published 8 articles in high-quality journals, 1 book chapter and 1 book; all of these publications have undergone peer review. In addition there is 1 preprint that is currently being revised. I have involved 6 collaborators from Chile, Japan, Korea and The Netherlands. Furthermore, I have organized three international workshops on topological insulators were supported with the grant. Overall, very good progress was made on Projects 1 and 3. For Project 1 we had to correct an earlier result of ours, and take the real-valuedness constraint of electromagnetic waves properly into account. An unexpected exciting result concerning project 3 was the topological nature of surface waves at metal-dielectric interfaces; it featured a hitherto unknown bulk-boundary correspondence that has no hermitian analog. Unfortunately, the interplay between topology and non-linear effects (Project 2) has not been concluded.

研究分野：mathematical physics

キーワード：condensed matter　topological insulators　classical waves　non-hermitian

Max Lein

１．研究開始当初の背景 (original plan & background)

The original proposal was split into three sub projects.

The aim of **Project 1** was to find rigorous connections between Maxwell's equations for periodic media and tight-binding operators that possess the necessary symmetries and encapsulate the essential features of the continuum model in the relevant frequency range. The strategy was to develop a theory of pseudodifferential operators with symmetries. These would enter as the main technical tool in the second step where we would adapt earlier works by De Nittis and myself to derive effective tight-binding models with symmetries.

Project 2 focused on topological effects in non-linear media, including the justification of effective models like the non-linear Schrödinger equation. A particular focus was on weakly non-linear electromagnetic media. It builds on Project 1.

Project 3 was concerned with bulk-boundary correspondences in electromagnetic systems. Specifically, the aims were to find a proof for Haldane's bulk-boundary correspondence and search for other, hitherto unknown bulk-boundary correspondences. It builds on Project 1.

２．研究の目的 (aim)

The overarching goal was to gain a better understanding of quantum-wave analogies in general. Topological phenomena for classical waves are a specific case.

３．研究の方法 (methodology)

From the perspective of mathematics, the main tools used are functional analysis, the theory of partial differential equations, Bloch-Floquet theory, operator algebras, K-theory, the theory of Krein spaces, vector bundle theory and pseudodifferential theory. From the viewpoint of physics, the main tools were classification theory of topological insulators.

４．研究成果 (results)

Originally the plan for **Project 1** was to start from a classification result I obtained with Giuseppe De Nittis (Annals of Physics 350, 2014). After submission I met with Kostya Bliokh (a future collaborator) who pointed out a very subtle error in my earlier classification work, namely that complex conjugation should be regarded as a constraint rather than a symmetry. Its presence reflects the fact that electromagnetic waves (and other classical waves) are real-valued. That led to three works, one about how to make quantum-wave analogies rigorous (Annals of Physics 396, 2018), which explained how to correctly rewrite a class of first-order classical wave equations in the form of a Schrödinger equation; the second was a correct topological classification of dielectric, lossless electromagnetic media (Advances in Theoretical and Mathematical Physics 26, 2019). The third work was dedicated to showing the equivalence of the Schrödinger form of a classical wave equation and the classical second-order wave equation; the point of the work was to show that in photonic crystals Chern numbers computed in any of these formalisms must agree (Journal of Mathematical Physics 61, 2020); this work was selected as the Editor's Pick of the issue.

Our topological bulk classification of electromagnetic media (Annals of Physics 396, 2018) showed that the classes AIII and BDI were unphysical for electromagnetic media; this obviated much of Project 1 as planned in the proposal; instead our earlier result on effective dynamics (Communications in Mathematical Physics 332, 2014) applies verbatim.

Together with Giuseppe De Nittis I wrote a book on an analytic-algebraic approach to make linear response theory rigorous. Linear response theory is one of the central tools to link boundary currents to topological invariants. For periodic and random class A operators the Kubo formula for the conductivity coefficients reproduces the formula for the Chern numbers, which are the relevant topological invariants. What is special about our framework is that it applies to discrete and continuum operators alike and can deal with effects of disorder. In principle, it is applicable to photonic crsytals and other classical wave equations.

In summary, Project 1 was concluded after completion.

Project 2 was meant as a follow-up project after the completion of Project 1. However, the PI decided to prioritize Project 3 in view of new results. So currently, the persistence of topological effects in the presence of weak non-linearities and the correct form of non-linear equations in various regimes has not been proven.

Project 3 is split into two parts. The first is about proving the Haldane's bulk-edge correspondence starting from Maxwell's equations. Some partial results have been obtained, but ultimately a proof has not yet been completed. This is for two reasons: first of all, unlike periodic Schrödinger operators, after Bloch-Floquet decomposition Maxwell operators always have a discontinuity at $k = 0$ due to "ground state bands" with approximately linear dispersion around the origin. They are always present, because they correspond to waves with long wavelength that to leading order only see the unit cell average of electric permittivity and magnetic permeability. This leads to a technical problem when one wants to adapt the classification procedure for periodic quantum systems; the eigenspaces below the spectral gap do *not* form a vector bundle if one includes the singular point $k = 0$. Instead, one has to study extensions of vector bundles on $\mathbb{T}^2 \setminus 0$ to $\mathbb{T}^2$, which is work-in-progress. The second non-trivial complication concerns boundary conditions. Maxwell's equations in principle admit an infinite number of boundary conditions. This requires a more delicate approach to classifying the system with boundary, which is presently not available. Importantly, boundary conditions may break time-reversal symmetry, so I expect that the validity of bulk-boundary correspondence is not always true. This is consistent with recent literature.

The second line of investigation came in 2018 when together with my collaborators Konstantin Bliokh, Daniel Leykam and Franco Nori we found a new bulk-boundary correspondence that has no analog in selfadjoint (hermitian) sytems — including all quantum systems (Nature Communications 10, 2019). That is quite significant as it is one of the very few examples where the bulk-boundary correspondence is not derived from some hermitian system. It shifted the focus of the investigation from selfadjoint to *Krein*-selfadjoint (pseudo-hermitian) operators. A follow-up work in this direction with Koji Sato classified magnonic crystals that are described by pseudohermitian equations of motion (Physical Review B 100, 2019).

As part of that effort, I have written a preprint that explains how to classify non-selfadjoint operators; together with Vicente Lenz I am working on a revision. The main results are three-fold: first of all, it gives criteria from physics which select a meaningful topological bulk classification. Secondly, it extends known classification results by Kawabata et al. from periodic tight-binding operators to *spectral* operators; importantly, this includes certain classes of random operators. And lastly, it shows that there likely exists no generic classification theory of generic non-selfadjoint operators in infinite dimensions. This line of investigation is currently continued with the help of a Kiban C grant.

5．主な発表論文等

〔雑誌論文〕　計8件（うち査読付論文　8件／うち国際共著　8件／うちオープンアクセス　0件）

1．著者名 Giuseppe De Nittis and Max Lein	4．巻 61
2．論文標題 Equivalence of Electric, Magnetic and Electromagnetic Chern Numbers for Topological Photonic Crystals	5．発行年 2020年
3．雑誌名 Journal of Mathematical Physics	6．最初と最後の頁 22901
掲載論文のDOI（デジタルオブジェクト識別子） 10.1063/1.5094589	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Giuseppe De Nittis and Max Lein	4．巻 26
2．論文標題 Symmetry Classification of Topological Photonic Crystals	5．発行年 2020年
3．雑誌名 Advances in Theoretical and Mathematical Physics	6．最初と最後の頁 1467-1531
掲載論文のDOI（デジタルオブジェクト識別子） 10.4310/ATMP.2019.v23.n6.a2	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Max Lein and Koji Sato	4．巻 100
2．論文標題 The Krein-Schroedinger Formalism of Bosonic BdG and Certain Classical Systems and Their Topological Classification	5．発行年 2019年
3．雑誌名 Physical Review B	6．最初と最後の頁 75414
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevB.100.075414	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Konstantin Y. Bliokh, Daniel Leykam, Max Lein and Franco Nori	4．巻 December
2．論文標題 Topological Non-Hermitian Origin of Surface Electromagnetic and Acoustic Waves	5．発行年 2019年
3．雑誌名 Optics and Photonic News	6．最初と最後の頁 0
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Konstantin Bliokh, Daniel Leykam, Max Lein and Franco Nori	4．巻 10
2．論文標題 Topological non-Hermitian origin of surface Maxwell waves	5．発行年 2019年
3．雑誌名 Nature Communications	6．最初と最後の頁 1-7
掲載論文のDOI（デジタルオブジェクト識別子） 10.1038/s41467-019-08397-6	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Giuseppe De Nittis and Max Lein	4．巻 In Press
2．論文標題 The Schroedinger Formalism of Electromagnetism and Other Classical Waves — How to Make Quantum-Wave Analogies Rigorous	5．発行年 2018年
3．雑誌名 Annals of Physics	6．最初と最後の頁 1-49
掲載論文のDOI（デジタルオブジェクト識別子） 10.1016/j.aop.2018.02.019	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Max Lein	4．巻 In Press
2．論文標題 Taking Inspiration from Quantum-Light Analogies — Recent Results for Photonic Crystals	5．発行年 2018年
3．雑誌名 Macroscopic Limits of Quantum Systems — Munich, Germany, March 20-April 1, 2017	6．最初と最後の頁 1-28
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Giuseppe De Nittis and Max Lein	4．巻 18
2．論文標題 Derivation of Ray Optics Equations in Photonic Crystals via a Semiclassical Limit	5．発行年 2017年
3．雑誌名 Annales Henri Poincare	6．最初と最後の頁 1789-1831
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s00023-017-0552-7	査読の有無 有
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〔学会発表〕　計24件（うち招待講演　11件／うち国際学会　7件）

1．発表者名 Max Lein
2．発表標題 Linear Response Theory — An Analytic-Algebraic Approach
3．学会等名 Rigorous Statistical Mechanics and Related Topics（招待講演）
4．発表年 2019年

1．発表者名 Max Lein
2．発表標題 Deriving Bulk-Boundary Correspondences from Conserved Quantities — Topological Surface Modes at Metal-Dielectric Interfaces
3．学会等名 The 4th A3 Metamaterials Forum 2019（招待講演）
4．発表年 2019年

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2．発表標題 A Pedagogical Guide to the Theory of Topological Insulators
3．学会等名 University of Tokyo
4．発表年 2019年

１．発表者名
Max Lein

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Towards a Rigorous Proof of Haldane's Photonic Bulk-Edge Correspondence

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2018年

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１．発表者名
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２．発表標題
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３．学会等名
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2017年

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Max Lein

２．発表標題
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３．学会等名
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４．発表年
2017年

１．発表者名
Max Lein

２．発表標題
Rigorous Analogies Between Quantum Systems and Certain Wave Equations

３．学会等名
Spectral Theory of Novel Materials（招待講演）（国際学会）

４．発表年
2016年

１．発表者名 Max Lein
２．発表標題 Classification of Topological Insulators for Classical Light
３．学会等名 Topological States of Light and Beyond（招待講演）（国際学会）
４．発表年 2016年

１．発表者名 Max Lein
２．発表標題 “Semiclassical” Ray Optics in Photonic Crystals
３．学会等名 Seminar of the Faculty of Mathematics
４．発表年 2016年

１．発表者名 Max Lein
２．発表標題 Unraveling the Relationship Between Topology and Physics
３．学会等名 Seminar of the Faculty of Physics
４．発表年 2016年

１．発表者名 Max Lein
２．発表標題 Rigorous Analogies Between Quantum Systems and Classical Waves
３．学会等名 Physical and mathematical approaches to interacting particle systems -In honor of 70th birthday of Herbert Spohn-（招待講演）（国際学会）
４．発表年 2017年

〔図書〕　計1件

1．著者名	4．発行年
Giuseppe De Nittis and Max Lein	2017年
2．出版社	**5．総ページ数**
Springer	136
3．書名	
Linear Response Theory - An Analytic-Algebraic Approach	

〔産業財産権〕

〔その他〕

https://www.maxlein.com
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https://maxlein.com/research/topics/
Max Lein's website
http://maxlein.com/research/topics/

6．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考

7．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計3件

国際研究集会	開催年
Topological Phenomena in Non-hermitian and Non-equilibrium Systems	2020年～2020年
国際研究集会	**開催年**
Recent Progress in Mathematics of Topological Insulators	2019年～2019年
国際研究集会	**開催年**
Progress in the Mathematics of Topological States of Matter	2018年～2018年

8．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関

オランダ	Technical University Delft			