様　式　C－１９、F－１９－１、Z－１９　（共通）

科学研究費助成事業　　研究成果報告書

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研究課題名（和文）Thermodynamic formalism for non-compact spaces with applications in conformal dynamics

研究課題名（英文）Thermodynamic formalism for non-compact spaces with applications in conformal dynamics

研究代表者

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研究者番号：９０７４１８６９

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研究成果の概要（和文）：可算無限シンボルを持つ位相的マルコフ連鎖と非一様拡大的な等角力学系に対して、エルゴード理論や熱力学形式を研究した。例として、区間上のマルコフ写像と尖点を持つ双曲曲面上の測地流が挙げられる。力学系的性質と不変集合のフラクタル幾何学的性質の相互作用を中心にした。マルチフラクタル解析はバーコフ平均、cusp windingsなどに対して行われた。

研究成果の学術的意義や社会的意義
Chaos in dynamical systems and the intricate geometry of fractal sets are fascinating for everyone. These phenomena have resulted in mathematical research of highest level over the past 50 years. In our project, we worked on some of the recent trends in these fields.

研究成果の概要（英文）：We investigated ergodic theory and thermodynamic formalism for topological Markov chains with countably many symbols, and non-uniformly expanding conformal dynamical systems. Examples include Markov interval maps and geodesic flows on hyperbolic surfaces with cusps. Our main focus was the interplay of dynamics and fractal-geometry of associated invariant sets. We performed a multifractal analysis of Birkhoff averages, cusp windings, local dimensions of Gibbs measures, and harmonic functions.

研究分野：Ergodic theory, Fractal geometry

キーワード：Ergodic Theory　Dynamical Systems　Fractal Geometry　Random walks　Multifractal analysis

１．研究開始当初の背景

Thermodynamic formalism is a branch of ergodic theory originating from statistical physics developed in the 1970s by Sinai, Ruelle and Bowen. The aim is to characterize meaningful invariant measures for a given topological dynamical system. It is well known that for uniformly expanding conformal dynamical systems there is fruitful interplay between thermodynamic formalism and the fractal geometry of associated limit sets.

At the end of the 1990s, the well-known thermodynamic formalism for finite-state Markov chains was extended to certain infinite-state Markov chains satisfying strong mixing assumptions. Recent research aims to develop the formalism under weaker mixing assumptions, and to apply the thermodynamic formalism to non-uniformly expanding conformal dynamical systems.

Ergodic theory often allows us to describe the typical long-term behaviour of a chaotic dynamical system with respect to an invariant probability measure. On the other hand, multifractal analysis aims to determine a complete spectrum of all possible exceptional behaviours by means of fractal geometry. Multifractal analysis is often based on thermodynamic formalism. Recent research deals with multifractal analysis for non-uniformly expanding conformal dynamical systems.

２．研究の目的

The aim of this project was to find new phenomena and to develop new theory on each of the following two fields (A), (B), as well as their interplay:
(A) Thermodynamic formalism for countable state Markov chains.
(B) Fractal geometry of non-uniformly expanding conformal dynamical systems.
In particular, it should be investigated to what extent (A) differs from the theory for finite-state Markov chains, and the fractal geometry in (B) differs from the uniformly expanding setting.

３．研究の方法

The method of inducing is an extremely powerful tool to deal with non-uniformly expanding conformal dynamical systems. This method was used to deal with parabolic generators in [2]. In [3] we established a framework guaranteeing the existence of a nice inducing scheme.

In [1], we used multifractal analysis to study the regularity of harmonic functions. The underlying multifractal formalism was further extended in [4] to make it applicable to conjugacy maps of conformal iterated function systems. In [5] we used ideas from Patterson-Sullivan theory to study growth functions of normal subgroups of free groups. Some of the main results in [6] rely on recurrence criteria for random walks on the real line.

４．研究成果

Outline

We investigated ergodic theory and thermodynamic formalism for topological Markov chain with countably many symbols, and non-uniformly expanding conformal dynamical systems. Examples include Markov interval maps and geodesic flows on hyperbolic surfaces with cusps. Our main focus was the interplay of dynamics and fractal-geometry of the associated limit sets. We performed multifractal analysis of Birkhoff averages, cusp windings, local dimensions of Gibbs measures, and harmonic functions.

Publications

[1] J. Jaerisch, H. Sumi: *Spectral Gap Property for Random Dynamics on the Real Line and Multifractal Analysis of Generalised Takagi Functions*,
Communcations in Mathematical Physics, volume 377, 2020.

[2] J. Jaerisch, M. Kesseboehmer, and S. Munday: *A multifractal analysis for cuspidal windings on hyperbolic surfaces*
Stochastics and Dynamics, Online Ready, 2021.

[3] J. Jaerisch, H. Takahasi: *Mixed multifractal spectra of Birkhoff averages for non-uniformly expanding one-dimensional Markov maps with countably many branches,*
Advances in Mathematics, volume 385, 2021.

[4] J. Jaerisch, H. Sumi: *Multifractal Formalism for generalised local dimension spectra of Gibbs measures on the real line,*
Journal of Mathematical Analysis and Applications, Volume 491, Issue 2, 2020.

[5] J. Jaerisch, K. Matsuzaki: *Weighted cogrowth formula for free groups*,
Groups, Geometry and Dynamics, Volume 14, Issue 2, 2020.

[6] M. Groeger, J. Jaerisch, M. Kesseboehmer: *Thermodynamic formalism for transient dynamics on the real line*,
Preprint on arXiv, arxiv.org/abs/1905.09077.

Let us now discuss our main results in more detail.

In [1] we studied the i.i.d. random iteration of finitely many expanding diffeomorphisms on the real line without a common fixed point, giving rise to a transient random walk on the real line. By establishing the spectral gap property of the associated transition operator acting on Hölder spaces, we show the Hölder continuity of harmonic functions for the random walk and introduce generalised Takagi functions. We perform a complete multifractal analysis of the pointwise Hölder exponents of these functions. This is a joint work Hiroki Sumi (Kyoto University) published in [1].

In [4] we obtained new results on the Hölder regularity of various fractal functions, such as distribution functions and conjugacy maps associated with conformal iterated function systems on the real line. In order to do so, we developed the multifractal formalism for generalized local dimension of a Gibbs measure supported on the attractor of a conformal iterated functions system on the real line. This is a joint work Hiroki Sumi (Kyoto University) published in [4].

In [2] we investigated multifractal decompositions of the limit set of a finitely generated, free Fuchsian group with parabolic generators with respect to the mean cusp-winding number. We completely determine its multifractal spectrum by means of a certain free energy function. This allows us to study the asymptotic behaviour of the cusp winding process for the geodesic flow on the associated hyperbolic surface. To establish our results, we generalize previously obtained results on the multifractal formalism for infinite iterated function systems to the setting of infinite graph directed Markov systems. This is a joint work with Sara Munday (John Cabot University, Italy) and Marc Kesseboehmer (University Bremen, Germany) published in [2].

The following has been obtained in [3]. For a Markov map of an interval or the circle with countably many branches and finitely many neutral periodic points, we establish conditional variational formulas for the mixed multifractal spectra of Birkhoff averages of countably many observables, in terms of the Hausdorff dimension of invariant probability measures. This is a joint work with Hiroki Takahasi (Keio University) published in [3].

In [5] we investigate the relationship between geometric, analytic and probabilistic indices for quotients of the Cayley graph of the free group. We verified an extension of Grigorchuk's cogrowth formula. This is a joint work with Katsuhiko Matsuzaki (Waseda University) published in [5].

In our preprint [6] we develop a new thermodynamic formalism to investigate the transient behaviour of maps on the real line which are skew-periodic $\mathbb{Z}$-extensions of expanding interval maps. Our main focus lies in the dimensional analysis of the recurrent and transient sets as well as in determining the whole dimension spectrum with respect to escaping sets. Our results provide a one-dimensional model for the phenomenon of a dimension gap occurring for limit sets of Kleinian groups. This is a joint work with Marc Kesseboehmer (University Bremen, Germany) and Maik Groeger (Jagiellonian University, Poland).

５．主な発表論文等

〔雑誌論文〕　計5件（うち査読付論文　3件／うち国際共著　3件／うちオープンアクセス　0件）

１．著者名 Johannes Jaerisch and Hiroki Takahasi	４．巻 385
２．論文標題 Mixed multifractal spectra of Birkhoff averages for non-uniformly expanding one-dimensional Markov maps with countably many branches	５．発行年 2021年
３．雑誌名 Advances in Mathematics	６．最初と最後の頁 1--45
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 Johannes Jaerisch and Hiroki Sumi	４．巻 377
２．論文標題 Spectral Gap Property for Random Dynamics on the Real Line and Multifractal Analysis of Generalised Takagi Functions	５．発行年 2020年
３．雑誌名 COMMUNICATIONS IN MATHEMATICAL PHYSICS	６．最初と最後の頁 1--36
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 Johannes Jaerisch and Katsuhiko Matsuzaki	４．巻 14
２．論文標題 Weighted cogrowth formula for free groups	５．発行年 2020年
３．雑誌名 GROUPS GEOMETRY AND DYNAMICS	６．最初と最後の頁 349--368
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 Jaerisch Johannes、Kesseb?hmer Marc、Munday Sara	４．巻 1
２．論文標題 A multifractal analysis for cuspidal windings on hyperbolic surfaces	５．発行年 2021年
３．雑誌名 Stochastics and Dynamics	６．最初と最後の頁 2140007～2140007
掲載論文のDOI（デジタルオブジェクト識別子） 10.1142/S0219493721400074	査読の有無 無
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 －

1．著者名 Jaerisch Johannes、Sumi Hiroki	4．巻 491
2．論文標題 Multifractal formalism for generalised local dimension spectra of Gibbs measures on the real line	5．発行年 2020年
3．雑誌名 Journal of Mathematical Analysis and Applications	6．最初と最後の頁 124246～124246

掲載論文のDOI（デジタルオブジェクト識別子） 10.1016/j.jmaa.2020.124246	査読の有無 無
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 －

〔学会発表〕　計21件（うち招待講演　11件／うち国際学会　13件）

1．発表者名 Johannes Jaerisch and Hiroki Takahasi
2．発表標題 Mixed Birkhoff spectra of one-dimensional Markov maps
3．学会等名 MSJ Spring Meeting, Statistics and Probability Session
4．発表年 2021年

1．発表者名 Johannes Jaerisch and Hiroki Takahasi
2．発表標題 Multifractal analysis of Birkhoff averages for non-uniformly expanding Markov interval maps Invited
3．学会等名 Kyoto Dynamics Seminar（招待講演）
4．発表年 2020年

1．発表者名 Johannes Jaerisch
2．発表標題 Multifractal analysis of cuspidal windings on hyperbolic surfaces
3．学会等名 RIMS Online Workshop “Complex Dynamics and Related Topics（招待講演）（国際学会）
4．発表年 2020年

1．発表者名 Johannes Jaerisch and Hiroki Takahasi
2．発表標題 Mixed Birkhoff spectra of one-dimensional Markov maps
3．学会等名 RIMS 研究集会「数理科学の諸問題と力学系理論の新展開」
4．発表年 2020年

1．発表者名 Johannes Jaerisch
2．発表標題 Multifractal analysis of dynamical systems
3．学会等名 名古屋大学談話会（招待講演）
4．発表年 2020年

1．発表者名 Johannes Jaerisch
2．発表標題 Dimension gaps in transient dynamics on the real line
3．学会等名 力学系-新たな理論と応用に向けて，京都大 学（国際学会）
4．発表年 2019年～2020年

1．発表者名 Johannes Jaerisch
2．発表標題 Spectral gap property for random dynamics on the real line and multifractal analysis of generalised Takagi functions
3．学会等名 Research on the Theory of Random Dynamical Systems and Fractal Geometry, Kyoto University（国際学会）
4．発表年 2019年～2020年

1．発表者名
Johannes Jaerisch

2．発表標題
Multifractal formalism for generalised local dimension spectra of Gibbs measures on the real line

3．学会等名
2019年度秋季総合分科会　金沢大学

4．発表年
2019年～2020年

1．発表者名
Johannes Jaerisch

2．発表標題
Recurrent and transient sets for skew-periodic interval maps

3．学会等名
エルゴード理論とその周辺，長岡

4．発表年
2019年～2020年

1．発表者名
J. Jaerisch

2．発表標題
Thermodynamic formalism for transient dynamics on the real line

3．学会等名
力学系 -理論と応用の融合、京都大学（国際学会）

4．発表年
2018年

1．発表者名
J. Jaerisch

2．発表標題
Spectral gap property for random dynamics on the real line and multifractal analysis of gener-alised Takagi functions

3．学会等名
AIMS Conference on Dynamical Systems, Differential Equations and Applications, National Taiwan University（招待講演）（国際学会）

4．発表年
2018年

１．発表者名
J. Jaerisch

２．発表標題
Weighted cogrowth formula for free groups

３．学会等名
Conference on Ergodic theory, Osaka University（国際学会）

４．発表年
2018年

１．発表者名
J. Jaerisch

２．発表標題
Thermodynamic formalism for transient dynamics on the real line

３．学会等名
Complex Dynamics conference, Kyoto University（招待講演）（国際学会）

４．発表年
2018年

１．発表者名
J. Jaerisch

２．発表標題
Thermodynamic formalism for transient dynamics on the real line

３．学会等名
Colloquium at University North Texas, TX, US（招待講演）（国際学会）

４．発表年
2018年

１．発表者名
J. Jaerisch

２．発表標題
Weighted cogrowth formula for free groups

３．学会等名
日本数学会2019年度年会，東京工業大学 Geometry session

４．発表年
2019年

１．発表者名
J. Jaerisch

２．発表標題
Dimension gaps in transient dynamics on the real line

３．学会等名
日本数学会2019年度年会，東京工業大学 Statistics and probability session

４．発表年
2019年

１．発表者名
Johannes Jaerisch

２．発表標題
Thermodynamic Formalism for infinitely generated hyperbolic conformal systems I

３．学会等名
エルゴード理論の最近の発展，RIMS Kyoto Univ.（招待講演）（国際学会）

４．発表年
2017年

１．発表者名
Johannes Jaerisch

２．発表標題
Thermodynamic Formalism for infinitely generated hyperbolic conformal systems II

３．学会等名
エルゴード理論の最近の発展，RIMS Kyoto Univ.（招待講演）（国際学会）

４．発表年
2017年

１．発表者名
Johannes Jaerisch

２．発表標題
Thermodynamic formalism for non-compact spaces with applications to conformal dynamical systems I

３．学会等名
エルゴード理論とその周辺，広島大学（招待講演）（国際学会）

４．発表年
2017年

１．発表者名 Johannes Jaerisch
２．発表標題 Thermodynamic formalism for non-compact spaces with applications to conformal dynamical systems II
３．学会等名 エルゴード理論とその周辺，広島大学（招待講演）（国際学会）
４．発表年 2017年

１．発表者名 Johannes Jaerisch
２．発表標題 Thermodynamic formalism for non-compact spaces with applications to conformal dynamical systems III
３．学会等名 エルゴード理論とその周辺，広島大学（招待講演）（国際学会）
４．発表年 2017年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

Information on my research http://www.math.nagoya-u.ac.jp/~jaerisch/ Webpage of J. Jaerisch

６．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考

７．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計0件

８．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関