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研究課題名（英文）Stability, Global Galois Representations and Non-Abelian Zeta Functions

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１．研究計画の概要

This is a part of our researches on what we call Geometric Arithmetic. This consists of three main parts, namely, one for non-abelian class field theory, CFT for short, one for non-abelian zeta functions and one for geo-arithmetical intersection and cohomology, and the Rimann Hypothesis. In more details, in the part of CFT, we are going to see how our works on a general CFT for function fields over complex numbers, using the natural correspondence between irreducible unitary representations of fundamental groups of punctured Riemann surfaces and stable parabolic bundles of parabolic degree zero, can be worked out for other type of fields, particular for p-adic number fields and algebraic number fields; in the part of non-abelain zeta functions, we are going to introduce genuine non-abelain zetas for global fields using moduli spaces of stable objects and study their properties. As for the final parts about the Riemann hypothesis, we are going to investigate not only the classical RH but that for new zetas we introduced.

２．研究の進捗状況

Excellent progresses have been achieved. In part one, we are able to establish a general CFT for function fields over complex numbers. And motivated by this, we put forward a program in which stability is used to make a good selection among too complicated algebraic structures certain portions which can be controlled. In particular, we introduce a notion called omega-structures to measure ramifications for extension of p-adic number fields, an arithmetic analogue of parabolic structure of Seshadri, from which based on Fontaine's p-adic Galois representation theory, we obtained a conjectural micro reciprocity relating de Rham representations and what we call Semi-stable filtered (phi,N)-modules of slope zero, from which a non-abelain CFT for p-adic number fields can be established. As for non-abelian zeta functions, not only we are able to introduce natural non-abelian zeta for global fields, we worked out in details rank two zeta for which the RH can be established, but a far more general abelian zeta is introduced for any pair of (reductive group, maximal parabolic subgroup) defined over number fields. These new abelain zeta, when the pair about coincides with (special linear group, maximal parabolic corresponding to partition n=(n-1)+1), yields essentially the abelian part of non-abelian zetas
Moreover, we expect that the RH holds for all these abelian zetas This latest assertion is supported by many non-trivial numerical calculations, and indeed holds for SL(2,3),

Sp(4) and G_2 .

３．現在までの達成度
Very satisfied and many important results and constructions have been obtained.

４．今後の研究の推進方策
We are going to do some detailed analysis on our Conjectural Micro Reciprocity Law and give detailed calculations for the zeros of our zetas hoping to find some hidden laws for the distributions of zeros and possibly applications

５． 代表的な研究成果
（研究代表者、研究分担者及び連携研究者には下線）

〔雑誌論文〕（計 6 件）
①Lin Weng, A Rank Two Zeta and its Zeros, J Ramanujan Math. Soc, 21(3), 205-266, 2006 年, 有

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④K. Obitsu, W.K. To & L. Weng, The asymptotic behavior of the Takhtajan-Zograf metric, Comm. Math Phys, 284(2008), no1, 227-261, 有

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①L Weng, General Class Field Theory, Euler 300: Arithmetic Geometry, 2007 年 6 月,St Petersberg, Russia
②L Weng, General Class Field Theory, Algebraic and arithmetic structures of moduli
Spaces, 2007 年 9 月,Sapporo, Japan
③L Weng, Stability in arithmetic, Complex geometry in Osaka, 2007 年 11 月,Osaka, Japan
④L. Weng, Symmetries and the Riemann Hypothesis, Zeta functions in arithmetic and geometry, 2008 年 6 月, Muenster, Germany
⑤L. Weng, Symmetries and the Riemann Hypothesis, Algebraic geometry and its new developments, 2008 年 11 月, Math Dept, Kyoto U, Japan
⑥L. Weng, Stability and Arithmetic, 2008 年 12 月, Algebraic Number Theory and Its related Topics, RIMS, Kyoto U, Japan

〔図書〕（計 2 件）

① Lin Weng, Iku Nakamura ed, Arithmetic Geometry and Number Theory, World Scientific, 2006:
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② Lin Weng, Masanobu Kaneko ed. Conference on L-functions, 2007
(*) L. Weng: A geometric Approach to L-Functions, pp. 219-370

〔産業財産権〕
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〔その他〕