様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９　（共通）

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研究課題名（和文）The tt* equations: a bridge between the differential geometry of moduli spaces and classical isomonodromy theory

研究課題名（英文）The tt* equations: a bridge between the differential geometry of moduli spaces and classical isomonodromy theory

研究代表者

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研究成果の概要（和文）：本研究課題の下、tt*-戸田方程式に関して新しい結果が得られ、超対称性の理論的側面についてより深い理解に達した。リー群G = SL(n,C)に対するtt*-戸田方程式のC*上の解がすべて得られ、その漸近データとモノドロミーデータがすべて求まった。一般の単純リー群Gに対しては、無限遠における漸近データをストークスデータとの関係と共に、リー理論の概念を用いて解明した。また、このストークスデータの物理的解釈を見いだした。対外活動については、セミナー、ワークショップ、研究集会などを共同研究者と共に行なった。その結果、古典的手法を用いる研究者と現代的手法を用いる研究者との間の交流を促進することが出来た。

研究成果の学術的意義や社会的意義
The research results obtained during this project were published in international scientific journals. They were also made publicly available at https://arxiv.org/. They contributed to an active area of mathematical research related to physics, and to the training of young researchers.

研究成果の概要（英文）：The research activities of this project led to several new mathematical results on the tt*-Toda equations, and to a deeper understanding of theoretical aspects of supersymmetry. All solutions on C* of the tt*-Toda equations for the Lie group G = SL(n,C) were found, and all their asymptotic data and monodromy data were computed. For any complex simple Lie group G, the asymptotic data of solutions at infinity, and their relation to Stokes data, was elucidated in Lie-theoretic terms. A physical interpretation of this data was found. Activities such as lecture series, seminars, workshops, and conferences were carried out jointly with Co-Investigators. These activities facilitated the interaction of researchers in both classical and modern methods.

研究分野：数物系科学，微分幾何学

キーワード：Integrable systems　Quantum cohomology　tt* equations　Isomonodromy

様　式　Ｃ－１９、Ｆ－１９－１（共通）

１．研究開始当初の背景

Since the 1980's, many important developments in geometry were influenced by developments in theoretical physics such as Yang-Mills theory, conformal field theory, string theory, supersymmetry, and mirror symmetry. These developments have revealed deep and unexpected connections within mathematics itself, linking differential geometry, symplectic geometry, and algebraic geometry. In most cases the central concept is a nonlinear differential equation with special solutions (conjecturally) related to geometry.

The tt* equations (topological-antitopological fusion equations) were introduced by the theoretical physicists Cecotti and Vafa in the 1990's. They are related to geometry (harmonic bundles, quantum cohomology, Frobenius manifolds) and they may be studied by integrable systems methods (loop groups, isomonodromy theory). They exhibit interesting structures at the heart of modern geometry as well as nontrivial classical o.d.e. theory (Stokes Phenomenon, WKB theory, theory of Painleve equations).

This project has investigated the geometry of the tt* equations by combining classical and modern methods. The tt* equations span 3 rather different areas of mathematics: (i) nonlinear p.d.e. theory (especially the theory of integrable systems), (ii) geometry (Lie group theory, differential geometry, and moduli spaces of special solutions), (iii) classical isomonodromy theory (Painleve equations, the Stokes Phenomenon). This project has used combinations of all of these methods and ideas, in the particular case of the tt*-Toda equations, in order to obtain a complete understanding of those solutions which are relevant to physics.

２．研究の目的

The general aim of this project was to study the tt* equations and related equations, and the existence and properties of special solutions related to quantum cohomology/Frobenius manifolds, in joint research projects with researchers in Japan and abroad, by carrying out individual research projects and organizing seminars, workshops, and international conferences.

The specific aim was to extend the previous joint work of the Principal Investigator with Claus Hertling (Mannheim, Germany), Nan-Kuo Ho (NTHU, Taiwan), Alexander Its (IUPUI, USA), and Chang-Shou Lin (NTU, Taiwan) on the tt*-Toda equations, i.e. the tt* equations of Toda type.

An important aspect of this was to focus on the relations between (finite and infinite dimensional) Lie theory and the isomonodromy theory of differential equations (Stokes matrices and connection problems), and their relations with theoretical physics. It was expected that links with other areas of geometry, both classical and modern, would be revealed in the process of carrying out this program.

３．研究の方法

The project involves several sub-projects.

Sub-project 1: Generalization of previous joint work with Its and Lin on the tt*-Toda equations to any complex simple Lie group G.

Sub-project 2: Clarification of previous joint work with Hertling by using the loop group L SL(2,C) and its action on the infinite-dimensional Grassmannian.

Sub-project 3: Symplectic aspects of the tt*-Toda equations, and their applications.

Sub-project 4: Applications to differential geometry.

Sub-project 5: The tt* equations beyond Toda type.

Sub-project 6: Asymptotics and representation theory.

４．研究成果

During this 5 year project, the most substantial progress was made with subprojects 1,5. Progress was also made with aspects of subprojects 3,4,6.

Regarding subproject 1, the third in the series of publications "Isomonodromy aspects of the tt* equations of Cecotti and Vafa I,II,III" (M. A. Guest, A. R.Its, and C.-S. Lin) was completed during Year 1 of the project. This series solved the tt*-Toda equations on the punctured plane C* in the case G=SL(4,C), and computed explicitly the corresponding asymptotic data at zero and infinity, and also the monodromy data (Stokes data and connection matrix data). First, classical p.d.e. techniques were used to establish the existence and uniqueness of solutions in terms of their boundary conditions. Then, loop group techniques were used to related the asymptotic data at zero to the monodromy data. Finally, Riemann-Hilbert techniques were used to relate the monodromy data to the asymptotic data at infinity.

During Years 2-5 of the project considerable effort was devoted to generalising these techniques and results to the case G=SL(n,C) for any n. Although the strategy of the proofs was the same as in the case n=4, it was necessary to improve the efficiency of the methods, and this presented several technical difficulties. To resolve these difficulties, Lie-theoretic concepts were very useful in several places. Most of the proofs were completed by Year 5, and a preprint "The tt*-Toda equations of A_n type" (M. A. Guest, A. R.Its, and C.-S. Lin, arxiv:2302.04597) was completed in that year.

In order to treat the tt*-Toda equations for other Lie groups G, new ideas were needed. In previous joint work with Ho, the foundational step had been made by expressing the Stokes data in Lie-theoretic terms the case G=SL(n,C). This was generalised to the case of any simple Lie group G in the publication "Kostant, Steinberg, and the Stokes matrices of the tt*-Toda equations" (M. A. Guest, N.-K.Ho). In that work, the Stokes data was represented efficiently by an element of a Steinberg cross-section of the space of conjugacy classes of G. This cross-section is an affine space, composed of a finite number of root spaces of G. It was found (using work of Kostant) that the root spaces could be described in a version of the Coxeter Plane, a figure consisting of a finite number of points in a real 2-plane. Thus the Stokes data (part of the monodromy data) was quite well understood at the start of this project. Some progress was made with the (more difficult) problem of describing Lie-theoretically the connection matrix data (the remaining part), and this was reported in a plenary talk given at the 66th Geometry Symposium of the Mathematical Society of Japan in 2019. Work on this problem is continuing, with the aim of describing efficiently the entire monodromy data.

Substantial progress was made with the problem of generalizing the Riemann-Hilbert argument from the case G=SL(n,C) to the case of any simple Lie group G. This was based on the above description of the Stokes data in terms of the Coxeter Plane - rather surprisingly, the rays through the points in the Coxeter Plane provide a suitable contour for a Riemann-Hilbert which can be solved in an analogous way to that in the case G=SL(n,C). This gave rise to an efficient description of the asymptotics at infinity of solutions of the tt*-Toda equation. Furthermore, this description illuminated the role of the Stokes data in these asymptotics, thereby confirming and making precise the 1990's predictions of the physicists Dorey, Lerche, Vafa, and Warner that the Stokes data can be expected to describe "solitonic particles". In the context of the tt*-Toda equations, in the case where a solution corresponds to a physically realistic field theory, this also confirms a prediction of the physicists Cecotti and Vafa that the Stokes data enumerates BPS solitons. The publication "Topological-antitopological fusion and the quantum cohomology of Grassmannians" (M. A. Guest) reported these results.

In the same publication, progress on subproject 5 was reported. This was motivated by 1990's work of the physicist Bourdeau on (conjectural) solutions of the tt* equations corresponding to the quantum cohomology of the Grassmannian. In this case the tt*

equations are not of "Toda type". However, Bourdeau gave a physical argument to show that the these solutions were equivalent to a collection of solutions to the tt*-Toda equations which correspond to the quantum cohomology of projective space (to be more precise, the solution corresponding to the Grassmannian Gr_k(Cˆn) was predicted to be a re-arrangement of the solution corresponding to the projective space CPˆ(n-1)=Gr_1(Cˆn)). A mathematical explanation for this phenomenon was suggested in the above publication.

Another test case for subproject 5 was investigated by Tadashi Udagawa (PhD student at Waseda University). This was also motivated by 1990's work of the physicists Cecotti and Vafa, who predicted the existence of solutions of the tt* equations corresponding to the "fusion ring", an algebra constructed from representations of SU_n. In the case n=2, Udagawa showed that such solutions exist, and are constructible from solutions of the n=2 tt*-Toda equations. A preprint "Solutions of the tt*-equations constructed from the (SU_2)_k-fusion ring, and Smyth potentials" (T. Udagawa) was completed in Year 5 of the project. This research is also related to subproject 4 as one of the results was an explicit formula for the DPW potentials, in the sense of harmonic map theory and surface theory. Work on the case of the (SU_n)_k-fusion ring for n>2 is ongoing.

From the physical point of view, quantum cohomology rings and fusion rings are examples of chiral rings of a conformal field theory. Solutions of the tt* equations corresponding to such objects have the property that all Stokes matrices have integer entries. This property was investigated for solutions of the tt*-Toda equations by Yoshiki Kaneko (PhD student at Waseda University), and his results were published in "Solutions of the tt*-Toda equations from the quantum cohomology of minuscule flag manifolds" (Y. Kaneko, Nagoya Math. Jour., 2022), and "On some Lie-theoretic solutions of the tt*-Toda equations with integer Stokes data" (Y. Hateruma and Y. Kaneko, Jour. Phys. A, 2024).

Regarding subproject 3, some progress was made with the explicit construction of a symplectic form on the space of "local solutions" of the tt*-Toda equations, in joint projects with Nan-Kuo Ho (NTHU, Taiwan), and with Ryosuke Odoi (PhD student at Waseda University). The existence of a symplectic structure is expected from general results of Boalch, but it is of interest to construct it explicitly. In the case n=4, Odoi studied those solutions which are smooth in (some) neighbourhood of zero, and found a natural symplectic form together with canonical coordinates which can be written in terms of monodromy data, thus generalising a well known construction for the Third Painleve equation. He used this to solve the "constant problem" for global solutions, i.e. the problem of finding the constant which relates the asymptotics of the tau functions associated to the solution at zero and infinity. This was published as "Symplectic aspects of the tt*-Toda equations" (R. Odoi, Jour. Phys. A, 2022). It forms part of an ongoing joint project with Alexander Its (IUPUI, USA).

Regarding subproject 6, an unexpected advance was made in joint work with Takashi Otofuji (Nihon University). In this work, the Stokes data of the tt*-Toda equation (in the form of the Steinberg cross-section) was used to provide a mathematical explanation of an observation made by the physicists Fredrickson and Neitzke which related positive energy representations of the loop group to the holomorphic data (DPW potential) of a solution of the tt*-Toda equation. This was published as "Positive energy representations of affine algebras and Stokes matrices of the affine Toda equations" (M. A. Guest and T. Otofuji, Adv. Theor. Math. Phys. 2022). In related joint work with Katrin Wendland (UC Dublin, Ireland) the modular aspects of the classification of conformal field theories were studied. This project is ongoing.

Discussions with project members were essential for the work described above, and these often took place on the at lecture series, seminars, workshops, and conferences. The budget for this project was used mainly for partial support of such events. During Years 3 and 4 (2020-22) the worldwide Covid19 pandemic forced the cancellation or postponement of face-to-face activities. Some activities were held online, or in hybrid (face-to-face and online) format. The main events co-organised by members of the project were as follows (online or hybrid events are denoted by *):

(1) UK-Japan Winter School on Variational problems in Geometry and Mathematical Physics 7-10 January 2019, University of Leeds, UK. Foreign speakers included: P. Dorey, I. McIntosh, F. Burstall.
(2) The 2nd International Conference on Geometry of Submanifolds and Integrable Systems 22-26 March 2019, Osaka City University. Foreign speakers included: Fran Burstall, Udo Hertrich-Jeromin, Young Jin Suh, Peng Wang, Seong-Deog Yang.
(3) Workshop on Hamiltonian Systems and Lie theory 6 April 2019, Waseda University. Foreign speakers included: Peter Crooks.
(4) The 2nd Taiwan-Japan Joint Conference on Differential Geometry 1-5 November 2019, NCTS, National Taiwan University, Taiwan. Foreign speakers included: Chen-Yu Chi, Hung-Lin Chiu, Sheng-Fu Chiu, Ulrich Menne, Wei-Bo Su, Ryosuke Takahashi, Ye-Kai Wang, Siye Wu.
(5) The 3rd International Workshop "Geometry of Submanifolds and Integrable Systems, 2-6 December 2019, OCAMI, Osaka City University. Foreign speakers included: Luis Pedro Castellanos Moscoso, Josef Dorfmeister, Lynn Heller, Eduardo Mota Sánchez, Thomas Raujouan.
(6) Topics in the Geometry and Topology of Moduli Spaces 25 January 2020, Waseda University. Foreign speakers included: Florent Schaffhauser.
(7) Koriyama Geometry and Physics Days 2020 "Integrable systems, projective invariants, and related topics" Feb 8-10, 2020, Nihon University Koriyama.
(8) International Workshop on Geometric Evolution Equations and Related Fields 8-9 March 2021, OCAMI, Osaka City University*. Foreign speakers included: Jui-En Chang, Chih-Wei Chen, Siao-Hao Guo, Chun-Chi Lin, Yukihiro Seki, Wei-Bo Su, Chung-Jun Tsai, Chin-Tung Wu.
(9) The 27th Osaka City University International Academic Symposium: Mathematical Science of Visualization, and Deepening of Symmetry and Moduli 21-26 March 2021, OCAMI, Osaka City University*. Foreign speakers included: Ulrich Pinkall, Konrad Polthier.
(10) Toda equations, parabolic Higgs bundles, and related topics 5-6 October 2021, Waseda University*. Foreign speakers included: Georgios Kydonakis, Qiongling Li, Claudio Meneses, Florent Schaffhauser, Szilard Szabo.
(11) The 3rd Japan-Taiwan Joint Conference on Differential Geometry 1-3 November 2021, OCAMI, Osaka City University*. Foreign speakers included: Chin-Yu Hsiao, Adeel Ahmad Khan, Hsuan-Yi Liao, Hsueh-Yung Lin, Jesse Madnick, Chin-Lung Wang, Yi-Sheng Wang, Albert Wood.
(12) Koriyama Geometry and Physics Days 2021 "tt*-Toda equations and infinite-dimensional Lie algebras" Nov 14-15, 2021, Nihon University Koriyama.
(13) Special Geometry, Mirror Symmetry, and Integrable Systems 29 November - 2 December 2021, Waseda University*. Foreign speakers included: Murad Alim, Yalong Cao, Vicente Cortes, Liana David, Claus Hertling, Ian Strachan, Andrew Swann.
(14) The 4th International Workshop "Geometry of Submanifolds and Integrable Systems" 20-23 February 2022, OCAMI, Osaka City University*. Foreign speakers included: Fran Burstall, Emma Carberry, Robert Kusner, Katrin Leschke, Xiang Ma, Francisco Martín, Magdalena Toda, Martin Traizet.
(15) Differential Geometry and Integrable Systems (13th MSJ Seasonal Institute), 1-21 March 2022, Osaka City University*. Foreign speakers included: Fran Burstall, Josef Dorfmeister, Lynn Heller, Sebastian Heller, Nigel Hitchin, Robert Kusner, Katrin Leschke, Hui Ma, Franz Pedit, Ulrich Pinkall, Iskander Taimanov, Martin Traizet, Michael Wolf.
(16) Applications of Harmonic Maps and Higgs Bundles to Differential Geometry 28 May - 2 June 2022, RIMS Workshop (Type A), RIMS Kyoto. Foreign speakers included: Fran Burstall, John Loftin, Martin Traizet.
(17) Geometry, Stochastics & Dynamics: Celebrating 20 years of UK-Japan Winter Schools 12-16 September 2022, Imperial College London. Foreign speakers included: Kenji Fukaya, Chris Budd, Martin Hairer, Darryl Holm, Terry Lyons, Mark Pollicott, Graeme Segal, Peter Topping.
(18) Workshop on symplectic geometry and its applications 15 October 2022, Waseda University. Foreign speakers included: Eckhard Meinrenken.
(19) 2nd Shot of The 13th MSJ-SI "Differential Geometry and Integrable Systems" 26 November - 1 December 2022, Takamatsu. Foreign speakers included: Indranil Biswas, Emma Carberry, Shu-Cheng Chang, Robert Kusner, Franz Pedit, Mao-Pei Tsui.
(20) Online workshop on "Modular Forms in Geometry and Physics", 14 January 2023, Waseda University*. Foreign speakers included: Chang-Shou Lin.

５．主な発表論文等

〔雑誌論文〕　計22件（うち査読付論文　22件／うち国際共著　12件／うちオープンアクセス　21件）

１．著者名 Guest Martin、Ho Nan-Kuo	４．巻 13
２．論文標題 Polytopes, supersymmetry, and integrable systems	５．発行年 2021年
３．雑誌名 Josai Math. Monographs	６．最初と最後の頁 109～137
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Guest Martin	４．巻 74
２．論文標題 量子コホモロジー　-今なお有用だろうか-，数学	５．発行年 2022年
３．雑誌名 数学	６．最初と最後の頁 113～132
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

１．著者名 Hosono Shinobu、Lian Bong H、Yau Shing-Tung	４．巻 2021
２．論文標題 K3 surfaces from configurations of six lines in P2 and mirror symmetry II	５．発行年 2021年
３．雑誌名 International Mathematics Research Notices	６．最初と最後の頁 13231～13281
掲載論文のDOI（デジタルオブジェクト識別子） 10.1093/imrn/rnz259	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Coates Tom、Iritani Hiroshi	４．巻 61
２．論文標題 Gromov-Witten invariants of local P2 and modular forms	５．発行年 2021年
３．雑誌名 Kyoto Journal of Mathematics	６．最初と最後の頁 543～706
掲載論文のDOI（デジタルオブジェクト識別子） 10.1215/21562261-2021-0010	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Matsuhira Yuya、Nagoya Hajime	4．巻 64
2．論文標題 Connection Problem for the Generalized Hypergeometric Function	5．発行年 2021年
3．雑誌名 Funkcialaj Ekvacioj	6．最初と最後の頁 323～348
掲載論文のDOI（デジタルオブジェクト識別子） 10.1619/FESI.64.323	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

1．著者名 Yamakawa Daisuke	4．巻 58
2．論文標題 Quantization of simply-laced isomonodromy systems by the quantum spectral curve method	5．発行年 2022年
3．雑誌名 SUT Journal of Mathematics	6．最初と最後の頁 23～50
掲載論文のDOI（デジタルオブジェクト識別子） 10.55937/sut/1654147040	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

1．著者名 Homma Yasushi、Tomihisa Takuma	4．巻 60
2．論文標題 The spinor and tensor fields with higher spin on spaces of constant curvature	5．発行年 2021年
3．雑誌名 Annals of Global Analysis and Geometry	6．最初と最後の頁 829～861
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s10455-021-09791-4	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

1．著者名 Homma Yasushi、 Tomihisa Takuma	4．巻 31
2．論文標題 Spectra of the Rarita-Schwinger operator on some symmetric spaces.	5．発行年 2021年
3．雑誌名 J. Lie Theory	6．最初と最後の頁 249～264
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

1．著者名 Guest Martin A.	4．巻 16
2．論文標題 Topological-antitopological fusion and the quantum cohomology of Grassmannians	5．発行年 2021年
3．雑誌名 Japanese Journal of Mathematics	6．最初と最後の頁 155～183
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s11537-020-2036-7	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

1．著者名 Hosono Shinobu、Lian Bong H.、Takagi Hiromichi、Yau Shing-Tung	4．巻 14
2．論文標題 K3 surfaces from configurations of six lines in P2 and mirror symmetry I	5．発行年 2020年
3．雑誌名 Communications in Number Theory and Physics	6．最初と最後の頁 739～783
掲載論文のDOI（デジタルオブジェクト識別子） 10.4310/CNTP.2020.v14.n4.a2	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Hosono Shinobu、Takagi Hiromichi	4．巻 60
2．論文標題 Derived categories of Artin-Mumford double solids	5．発行年 2020年
3．雑誌名 Kyoto Journal of Mathematics	6．最初と最後の頁 107～177
掲載論文のDOI（デジタルオブジェクト識別子） 10.1215/21562261-2019-0036	査読の有無 有
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1．著者名 Dorfmeister Josef F.、Kobayashi Shimpei	4．巻 200
2．論文標題 Timelike minimal Lagrangian surfaces in the indefinite complex hyperbolic two-space	5．発行年 2020年
3．雑誌名 Annali di Matematica Pura ed Applicata (1923 -)	6．最初と最後の頁 521～546
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s10231-020-01005-1	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Martin Guest, Nan-Kuo Ho	4．巻 25
2．論文標題 Kostant, Steinberg, and the Stokes matrices of the tt*-Toda equations	5．発行年 2019年
3．雑誌名 Selecta Math., published online	6．最初と最後の頁 -
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s00029-019-0494-7	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Martin Guest, A. R. Its, C.-S. Lin	4．巻 374
2．論文標題 Isomonodromy aspects of the tt* equations of Cecotti and Vafa III. Iwasawa factorization and asymptotics	5．発行年 2020年
3．雑誌名 Commun. Math. Phys	6．最初と最後の頁 923-973
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s00220-019-03559-5	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Yoshihiro Ohnita, Reiko Miyaoka	4．巻 6
2．論文標題 Lagrangian geometry of the Gauss images of isoparametric hypersurfaces in spheres	5．発行年 2019年
3．雑誌名 Complex Manifolds	6．最初と最後の頁 265-278
掲載論文のDOI（デジタルオブジェクト識別子） 10.1515/coma-2019-0016	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

1．著者名 Yoshihiro Ohnita	4．巻 6
2．論文標題 Minimal Maslov number of R-spaces canonically embedded in Einstein-Kahler C-spaces	5．発行年 2019年
3．雑誌名 Complex Manifolds	6．最初と最後の頁 303-319
掲載論文のDOI（デジタルオブジェクト識別子） 10.1515/coma-2019-0013	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

1．著者名 Shinobu Hosono, Bong H Lian, Shing-Tung Yau	4．巻 2021
2．論文標題 K3 surfaces from configurations of six lines in P2 and mirror symmetry II: λ_K3-functions	5．発行年 2019年
3．雑誌名 Int. Math. Res. Notices published online	6．最初と最後の頁 13231～13281
掲載論文のDOI（デジタルオブジェクト識別子） 10.1093/imrn/rnz259	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Hosono Shinobu, Takagi Hiromichi	4．巻 14
2．論文標題 Movable vs Monodromy Nilpotent Cones of Calabi-Yau Manifolds	5．発行年 2018年
3．雑誌名 Symmetry, Integrability and Geometry: Methods and Applications	6．最初と最後の頁 -
掲載論文のDOI（デジタルオブジェクト識別子） 10.3842/SIGMA.2018.039	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 -

1．著者名 Coates Tom, Iritani Hiroshi	4．巻 58
2．論文標題 A Fock sheaf for Givental quantization	5．発行年 2018年
3．雑誌名 Kyoto J. Math.	6．最初と最後の頁 695-864
掲載論文のDOI（デジタルオブジェクト識別子） 10.1215/21562261-2017-0036	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Coates Tom, Iritani Hiroshi, Jiang, Yunfeng	4．巻 329
2．論文標題 The crepant transformation conjecture for toric complete intersections	5．発行年 2018年
3．雑誌名 Adv. Math.	6．最初と最後の頁 1002-1087
掲載論文のDOI（デジタルオブジェクト識別子） 10.1016/j.aim.2017.11.017	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Lisovyy O, Nagoya H. Roussillon J.	4．巻 59
2．論文標題 Irregular conformal blocks and connection formulae for Painleve V functions	5．発行年 2018年
3．雑誌名 J. Math. Phys.	6．最初と最後の頁 -
掲載論文のDOI（デジタルオブジェクト識別子） 10.1063/1.5031841	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Abe Hiroyuki、Aldabergenov Yermek、Aoki Shuntaro、Ketov Sergei V.	4．巻 2018
2．論文標題 Massive vector multiplet with Dirac-Born-Infeld and new Fayet-Iliopoulos terms in supergravity	5．発行年 2018年
3．雑誌名 Journal of High Energy Physics	6．最初と最後の頁 -
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/JHEP09(2018)094	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

〔学会発表〕　計25件（うち招待講演　14件／うち国際学会　11件）

1．発表者名 Martin Guest
2．発表標題 Quantum cohomology: is it still relevant?
3．学会等名 Autumn Meeting of the Mathematical Society of Japan, Chiba University, online（招待講演）
4．発表年 2021年

1．発表者名 Martin Guest
2．発表標題 Topological-antitopological fusion and Higgs bundles
3．学会等名 Workshop on Toda equations, parabolic Higgs bundles, and related topics, Waseda University, online
4．発表年 2021年

１．発表者名
Martin Guest

２．発表標題
Asymptotics of solutions of the tt*-Toda equations and some relations with physics

３．学会等名
Applied Analysis Seminar, Waseda University, online

４．発表年
2021年

１．発表者名
Martin Guest

２．発表標題
Asymptotic data and Stokes data for the tt*-Toda equations, and some relations with physics

３．学会等名
Beijing-Novosibirsk Seminar on Geometry and Mathematical Physics, online（国際学会）

４．発表年
2022年

１．発表者名
Martin Guest

２．発表標題
Asymptotic data and Stokes data for the tt*-Toda equations, and some applications

３．学会等名
17th Algebra, Analysis and Geometry Seminar, Kagoshima University, online

４．発表年
2022年

１．発表者名
Martin Guest

２．発表標題
Hamiltonian aspects of the tt*-Toda equations

３．学会等名
Workshop on Hamiltonian systems and Lie groups, Waseda University

４．発表年
2019年

１．発表者名
Martin Guest

２．発表標題
The (enhanced) Coxeter Plane: an application of differential equations to Lie theory

３．学会等名
Colloquium, Tokyo University of Science（招待講演）

４．発表年
2019年

１．発表者名
Martin Guest

２．発表標題
The tt* equations: monodromy-preserving deformations (Painleve theory) and DPW (harmonic map theory)

３．学会等名
66th Geometry Symposium, Nagoya University（招待講演）

４．発表年
2019年

１．発表者名
Martin Guest

２．発表標題
A loop group approach to finding global solutions of the tt* equations

３．学会等名
Integrable Systems and Harmonic Maps, Technical University of Vienna（招待講演）（国際学会）

４．発表年
2019年

１．発表者名
Martin Guest

２．発表標題
Polytopes, supersymmetry, and integrable systems

３．学会等名
Colloquium (Indiana University Purdue University Indianapolis)（招待講演）

４．発表年
2020年

１．発表者名
Yoshihiro Ohnita

２．発表標題
Minimal Maslov number of R-spaces canonically embedded in Einstein-Kahler C-spaces

３．学会等名
Variational Problems and the Geometry of Submanifolds (Centre International de Rencontres Mathematiques, Luminy, France) (国際学会)

４．発表年
2019年

１．発表者名
Yoshihiro Ohnita

２．発表標題
Lagrangian geometry of the Gauss images of isoparametric hypersurfaces

３．学会等名
Workshop on Isoparametric Theory (Beijing Normal University, China) (国際学会)

４．発表年
2019年

１．発表者名
Yoshihiro Ohnita

２．発表標題
Minimal Maslov number of R-spaces canonically embedded in Einstein-Kahler C-spaces

３．学会等名
22nd International Workshop on Differential Geometry of Submanifolds in Symmetric Spaces and Related Problems (Kyungpook National University, Daegu, Korea) (国際学会)

４．発表年
2019年

１．発表者名
Shinobu Hosono

２．発表標題
Movable vs Monodromy nilpotent cones in mirror symmetry of Calabi-Yau manifolds

３．学会等名
International Workshop on Derived Categories and Related Topics (Sun Yat-sen University, Guangzhou, China) (国際学会)

４．発表年
2019年

１．発表者名
Shinobu Hosono

２．発表標題
K3 analogues of the elliptic lambda function from a double cover family of K3 surfaces

３．学会等名
Interaction Between Algebraic Geometry and QFT (Moscow Institute of Physics and Technology, Russia（国際学会）

４．発表年
2019年

１．発表者名
Guest Martin

２．発表標題
The tt*-Toda equations

３．学会等名
Yorkshire and Durham Geometry Day, Leeds University, UK（招待講演）（国際学会）

４．発表年
2018年

１．発表者名
Guest Martin

２．発表標題
The Coxeter Plane and the tt*-Toda equations

３．学会等名
Symplectic Geometry Seminar, ETH Zurich, Switzerland（招待講演）

４．発表年
2018年

１．発表者名
Guest Martin

２．発表標題
Some geometric applications of meromorphic o.d.e. with irregular singular points

３．学会等名
Pure Mathematics Colloquium, University of Hamburg, Germany（招待講演）

４．発表年
2018年

１．発表者名
Guest Martin

２．発表標題
The enhanced Coxeter Plane - an application of integrable systems to Lie groups

３．学会等名
The 5th workshop "Complex Geometry and Lie Groups", University of Florence, Italy（招待講演）（国際学会）

４．発表年
2018年

１．発表者名
Guest Martin

２．発表標題
The tt*-Toda equations

３．学会等名
Geometry and Topology Seminar, University of Stuttgart,Germany

４．発表年
2018年

１．発表者名
Guest Martin

２．発表標題
The enhanced Coxeter Plane

３．学会等名
Geometry Seminar, University of Mannheim, Germany（招待講演）

４．発表年
2018年

１．発表者名
Guest Martin

２．発表標題
Quantum cohomology and reality

３．学会等名
Symplectic Geometry Seminar, ETH Zurich, Switzerland（招待講演）

４．発表年
2018年

１．発表者名
Ohnita Yoshihiro

２．発表標題
Geometry of Harmonic Maps and Integrable System Approach

３．学会等名
Mini-Workshop on Geometry and Mathematical Science, Osaka City University（招待講演）（国際学会）

４．発表年
2018年

１．発表者名
大仁田義裕

２．発表標題
アインシュタイン-ケーラーC-空間に標準的に埋め込まれたR-空間の最小マスロフ数

３．学会等名
水戸幾何セミナー（招待講演）

４．発表年
2018年

１．発表者名
Ohnita Yoshihiro

２．発表標題
Minimal Maslov number of R-spaces canonically embedded in Einstein-Kaehler C-spaces

３．学会等名
Variational Problems in Geometry and Mathematical Physics, UK-Japan Winter School 2019, Leeds University（招待講演）（国際学会）

４．発表年
2019年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

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６．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考
研究分担者	細野　忍 (Hosono Shinobu) (60212198)	学習院大学・理学部・教授 (32606)	

6．研究組織（つづき）

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考
研究分担者	大仁田　義裕 (Ohnita Yoshihiro) (90183764)	大阪公立大学・数学研究所・特別研究員 (24405)	

7．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計7件

国際研究集会 Special Geometry, Mirror Symmetry, and Integrable Systems	開催年 2021年～2021年
国際研究集会 Differential Geometry and Integrable Systems	開催年 2022年～2022年
国際研究集会 The 27th Osaka City University International Academic Symposium: Mathematical Science of Visualization, and Deepening of Symmetry and Moduli	開催年 2021年～2021年
国際研究集会 The 2nd Taiwan-Japan Joint Conference on Differential Geometry 1-5 November 2019, NCTS, National Taiwan University, Taiwan.	開催年 2019年～2019年
国際研究集会 The 3rd International Workshop "Geometry of Submanifolds and Integrable Systems, 2-6 December 2019, OCAMI, Osaka City University	開催年 2019年～2019年
国際研究集会 Variational problems in Geometry and Mathematical Physics UK-Japan Winter School, University of Leeds, UK	開催年 2019年～2019年
国際研究集会 The 2nd International Conference on Geometry of Submanifolds and Integrable Systems, Osaka City University	開催年 2019年～2019年

8．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関			
その他の国・地域	National Taiwan University	National Tsing-Hua University (Taiwan)		
米国	IUPUI			
ドイツ	Mannheim University	Hannover University		