【Grant-in-Aid for Scientific Research (S)】

Broad Section B

Title of Project : Strategic research to construct motivic units using new symmetry

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Keyword : Number Theory, Arithmetic Geometry

【Purpose and Background of the Research】

Conjectures such as the Beilinson conjecture, Tamagawa Number Conjecture and Iwasawa main conjecture concerning the special values of Hasse-Weil L-functions associated to algebraic variety defined over number field is a central problem in Mathematics, especially in arithmetic geometry. The Birch and Swinnerton-Dyer Conjecture for elliptic curves is a special case of these conjectures.

When the algebraic variety is the multiplicative group, whose associated Hasse-Weil L-function is the classical Riemann Zeta function, the above conjectures were solved by effort of many mathematicians a predominant tool being a motivic unit called the cyclotomic element (cyclotomic unit). One reason that the proof of the conjectures in the other cases are notoriously difficult is that there are no known method to systematically construct motivic units which intrinsically contain information concerning both the special values of the L-function and arithmetic information of the algebraic variety.

The purpose of this research is to study the polylogarithm, which are motivic elements systematically constructed for the multiplicative group, elliptic curves, and more general higher dimensional abelian varieties, wtih an eye towards future construction of motivic units. A concrete goal is to study the polylogarithm for a certain algebraic torus, and attempt to relate its Hodge realization to special values of L-function relevant in this case.

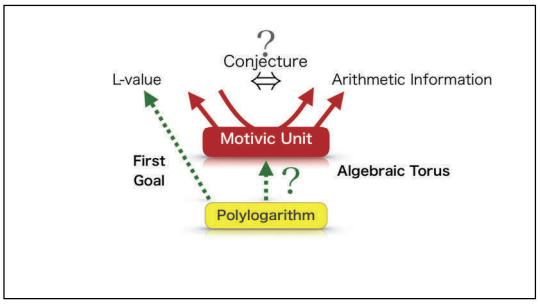

Figure 1 Concept

【Research Methods】

The research will be conducted by a team of experts including S. Yamamoto and S. Yasuda (Number Theory), S. Kobayashi (Iwasawa Theory), T. Terasoma (Motives), A. Shiho (p-adic theory) and T. Katsura (Operator Algebra). In particular, we will use techniques developed to explicitly represent higher degree cohomology classes, In order to obtain L-values from higher degree cohomology classes, it is necessary to use a theory of new symmetry called plectic structure developed by J. Nekovář and A. J. Scholl. We will first expand the theory of plectic structures for the Hodge case, and then will subsequently deal with the motivic, p-adic and étale cases.

【Expected Research Achievements and Scientific Significance】

If we succeed in explicitly determining the Hodge structure of the polylogarithm of the algebraic torus and succeed using the plectic structure to recover special values of the relevant L-function, then this would indicate the possibility that polylogarithm of the algebraic torus may be used to construct motivic units in this case.

【Publications Relevant to the Project】

- J. Nekovář and A. J. Scholl, *Introduction to plectic cohomology*, Advances in the theory of automorphic forms and their *L*-functions, Contemp. Math., vol. **664**, Amer. Math. Soc., Providence, RI, 2016, pp. 321–337.
- K. Bannai, K. Hagihara, S. Kobayashi, K. Yamada, S. Yamamoto, and S. Yasuda, *Category of mixed plectic Hodge structures* (2017), arXiv:1705.05522[math.AG].

【Term of Project】 FY2018-2022

【Budget Allocation】 91,900 Thousand Yen

【Homepage Address and Other Contact Information】

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