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研究代表者

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研究成果の概要(和文)：ヘンゼル離散付置環および算術スキームに関するヴェイ・エタールのコホモロジーに関する研究は、5つの論文につながりました。

(1) B. Morinとp進局所環上のスキームのBrauer群のp-およびl-corankに関する結果を証明しました。(2) B. Morinと、Pontrjagin双対性を満たす局所体上のヴェイ・エタールコホモロジー理論の定義を概説し、重み0で証明しました。(3) 上記の論文を用いて B. Morinと、局所体上の類体論の結果を証明しました。(4,5) 鈴木氏と、BDS予想のワイル・エタール版に関する研究を一般化しました。特に、織田氏の玉川数公式の新たな証明を得ました。

研究成果の学術的意義や社会的意義

Basic research does not have direct application, but contributes to the knowledge of humanity with applications in the future in mind. During the research students were involved and educated. Since my research involved an international collaboration, it also strengthens international understanding.

研究成果の概要(英文)：The research on Weil-étale cohomology for schemes over henselian discrete valuation rings and arithmetic schemes led to 5 publications, three in an international collaboration with B. Morin (France), and two with T. Suzuki.

(1) B. Morin and we proved a result regarding the p- and l-corank of the Brauer group of a smooth and proper scheme over a p-adic local ring, generalizing work of Colliot-Thelene, S. Saito, and Sato. (2) B. Morin and I outlined the definition of a Weil-étale cohomology theory for varieties over local fields which satisfy a Pontrjagin duality theory, and prove a duality result in weight zero. (3) B. Morin and I use the above to prove results on class field theory over local fields, generalizing and improving work of S. Saito and Yoshida. (4) T. Suzuki and I proved a Weil-étale version of the Birch and Swinnerton-Dyer conjecture for abelian varieties, and (5) generalized the result to one-motives. In particular, we obtain a new proof of the Tamagawa number formula of Oda.

研究分野：Motivic cohomology

キーワード：Brauer group Local fields Motivic cohomology Birch-Swinnerton-Dyer Class field theory

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1. 研究開始当初の背景

Motivic cohomology is an important invariant to study algebraic varieties X . For example, the Chow groups $CH_i(X)$, the group of units $\mathcal{O}(X)^\times$, the Picard group $\text{Pic}(X)$, and the Brauer group $\text{Br}(X)$ can be expressed in terms of motivic cohomology. Over finite fields, motivic cohomology groups (in its Weil-étale version) are fairly well understood: They are expected to be finitely generated, an integral model of l -adic and p -adic cohomology, and should satisfy duality in the category of finitely generated groups. In fact, this expectation is equivalent to the conjectures of Tate and Beilinson on algebraic cycles.

The next important cases to study are local fields and global fields of finite characteristic. For local fields, the groups to be finitely generated anymore. Indeed, the group of units $\mathcal{O}(X)^\times$ and the Picard group $\text{Pic}(X)$ are uncountable. However, they are extensions of finitely generated groups by pro- p -groups, and this is the starting point for our research.

Class field theory for varieties is an important topic, and it is related to motivic cohomology by duality. Class field theory is well understood over finite fields due to work of Kato-Saito, Schmidt-Szamuely, and Geisser-Schmidt. Over local fields, a theory was developed by Saito, Yoshida, and others. The situation is complicated by the fact that the fundamental group depends on the geometry of the special fiber of a model. This causes the reciprocity map to have a large kernel and cokernel, and its image is only the torsion part of the fundamental group.

2. 研究の目的

The main purpose of the current project was to construct motivic cohomology groups over local fields which are topological groups and satisfy some Pontrjagin duality, and give some applications to class field theory and Brauer groups over local fields. Our calculations with Brauer groups showed that locally compact groups cannot satisfy a duality, and that the cohomology groups need to be viewed as objects in the heart of the derived category of locally compact groups.

3. 研究の方法

Most of mathematical research is done with pencil, paper, and the library. But in order to exchange ideas, it is necessary to communicate with other experts in the field. Many ideas evolve in informal discussions with other mathematicians.

(1) Organization of international conferences

In order to be up to date and learn about the newest development, I used the grant to help organize a workshop on motives in Tokyo in 2019 and 2023 with S.Saito, T.Terasoma, and S.Kelly. Originally, this workshop was planned to happen yearly, but it was canceled in the years 2020-2022 due to travel restrictions. I am also a co-organizer with L.Hesselholt, M.Kerz, and H.Esnault of the tri-annual conference on algebraic K-theory and motivic cohomology in Oberwolfach, which was held in 2019 and 2022, and will be again held in 2025.

(2) Attendance of international and national conferences

Due to corona restrictions, international travel was limited, but I still attended one conference in St.Petersburg and several conferences in Japan, where I presented my work, listened to other researchers presentations, and discussed mathematics.

(3) Joint research projects with foreign institutions

Most of my work on this project was done in collaboration with B.Morin of Bordeaux University.

Due to the corona virus we had to conduct most of our discussions via email, but I visited him in 2021 and he visited me in 2023, which gave us a chance to work much more efficiently than by correspondence.

(4) International exchange

Besides my work with B.Morin, I visited several researchers at their institutes to discuss my research with them (for example in Szamuely in Budapest, Kolderup in Oslo, Binda in Milan, Szaumely in Pisa, and Tang in Taipei). During my sabbatical in 2021 I spend several months in Germany and gave presentations at 5 universities about my research.

4. 研究成果

(1) While calculating examples of motivic cohomology in weight one, B.Morin and I obtained results on the kernel of the Brauer-Manin pairing. More precisely, let \mathcal{X} be a regular scheme, flat and proper over the ring of integers \mathcal{O}_K of a p -adic field K , with generic fiber X and special fiber \mathcal{X}_s . Then $\text{Br}(\mathcal{X})$ is the left kernel of the Brauer-Manin pairing $\text{Br}(X) \times \text{CH}_0(X) \rightarrow \mathbb{Q}/\mathbb{Z}$. We showed that the kernel of the reduction map $\text{Br}(\mathcal{X}) \rightarrow \text{Br}(\mathcal{X}_s)$ is the direct sum of $(\mathbb{Q}/\mathbb{Z}[\frac{1}{p}])^s \oplus (\mathbb{Q}/\mathbb{Z})^t$ and a finite p -group, where $s + t = \rho_{\mathcal{X}_s} - \rho_X - I + 1$, for $\rho_{\mathcal{X}_s}$ and ρ_X the Picard numbers of \mathcal{X}_s and X , and I the number of irreducible components of \mathcal{X}_s . Moreover, $t > 0$ implies $s > 0$.

(2) The main achievement of his project (joint with B.Morin) was to define cohomological complexes of locally compact abelian groups associated with varieties over p -adic fields. In the special case where the weight is zero, we proved a duality theorem (under some weak assumption) in the form of a Pontryagin duality between locally compact motivic cohomology groups. We also gave an outline how our theory fits into a general conjectural framework, connecting with other cohomology groups over p -adic rings.

(3) As an application of the previous work, B.Morin and I gave applications to class field theory for smooth and proper schemes over p -adic fields. More precisely, let \mathcal{X} a regular scheme of dimension d , proper, flat, and geometrically irreducible over \mathcal{O}_K , and X be its generic fiber. Then under some mild assumptions on \mathcal{X} , there is a reciprocity isomorphism of locally compact groups $H_{ar}^{2d-1}(X, \mathbb{Z}(d)) \simeq \pi_1^{ab}(X)_W$ from the cohomology theory defined in above to an integral model $\pi_1^{ab}(X)_W$ of the abelianized fundamental group $\pi_1^{ab}(X)$. Moreover, after removing the contribution from the base field, the map becomes an isomorphism of finitely generated abelian groups.

(4) While working on Weil-étale cohomology over local fields, I discussed the situation over global fields of finite characteristic with Takashi Suzuki. This led to a Weil-étale version of the Birch and Swinnerton-Dyer conjecture. For example, the finiteness of the Tate-Shafarevich group is equivalent to the finiteness of Weil-étale cohomology groups.

(5) We extended this to one-motives by giving a formula for the leading coefficient at $s = 1$ of the L -function of one-motives over function fields in terms of Weil-étale cohomology. We also express the Tamagawa number of a torus introduced by Ono-Oesterlé in terms of Weil-étale cohomology, and reprove the Tamagawa number formula.

5. 主な発表論文等

〔雑誌論文〕 計7件（うち査読付論文 7件/うち国際共著 3件/うちオープンアクセス 0件）

1. 著者名 Geisser Thomas H., Suzuki Takashi	4. 巻 793
2. 論文標題 Special values of L-functions of one-motives over function fields	5. 発行年 2022年
3. 雑誌名 Journal für die reine und angewandte Mathematik (Crelles Journal)	6. 最初と最後の頁 281 ~ 304
掲載論文のDOI (デジタルオブジェクト識別子) 10.1515/crelle-2022-0081	査読の有無 有
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1. 著者名 Geisser Thomas H., Morin Baptiste	4. 巻 23
2. 論文標題 PONTRYAGIN DUALITY FOR VARIETIES OVER p-ADIC FIELDS	5. 発行年 2022年
3. 雑誌名 Journal of the Institute of Mathematics of Jussieu	6. 最初と最後の頁 425 ~ 462
掲載論文のDOI (デジタルオブジェクト識別子) 10.1017/S1474748022000469	査読の有無 有
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1. 著者名 Geisser Thomas H., Suzuki Takashi	4. 巻 208
2. 論文標題 A Weil-etale version of the Birch and Swinnerton-Dyer formula over function fields	5. 発行年 2020年
3. 雑誌名 Journal of Number Theory	6. 最初と最後の頁 367 ~ 389
掲載論文のDOI (デジタルオブジェクト識別子) 10.1016/j.jnt.2019.08.013	査読の有無 有
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1. 著者名 Geisser Thomas H., Morin Baptiste	4. 巻 260
2. 論文標題 On integral class field theory for varieties over p-adic fields	5. 発行年 2024年
3. 雑誌名 Journal of Number Theory	6. 最初と最後の頁 41 ~ 70
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1. 著者名 Geisser Thomas H., Morin Baptiste	4. 巻 238
2. 論文標題 On the kernel of the Brauer-Manin pairing	5. 発行年 2022年
3. 雑誌名 Journal of Number Theory	6. 最初と最後の頁 444 ~ 463
掲載論文のDOI (デジタルオブジェクト識別子) 10.1016/j.jnt.2021.09.003	査読の有無 有
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〔学会発表〕 計4件 (うち招待講演 4件 / うち国際学会 4件)

1. 発表者名 Thomas Geisser
2. 発表標題 Brauer groups and Neron-Severi groups of surfaces over finite fields
3. 学会等名 L-function and motives in Niseko (招待講演) (国際学会)
4. 発表年 2022年

1. 発表者名 T.Geisser
2. 発表標題 A Weil-etale version of the BSD conjecture
3. 学会等名 Conference of Motives in Tokyo (招待講演) (国際学会)
4. 発表年 2019年

1. 発表者名 T.Geisser
2. 発表標題 Relating Brauer groups and Tate-Shafarevich group
3. 学会等名 Arithmetic Algebraic Geometry in honor of T. Terasoma (招待講演) (国際学会)
4. 発表年 2019年

1. 発表者名 T.Geisser
2. 発表標題 Relating Brauer groups and Tate-Shafarevich group
3. 学会等名 Conference on Motives in St.Petersburg (招待講演) (国際学会)
4. 発表年 2018年

〔図書〕 計0件

〔産業財産権〕

〔その他〕

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6. 研究組織

	氏名 (ローマ字氏名) (研究者番号)	所属研究機関・部局・職 (機関番号)	備考
研究協力者	モリン バプティスト (Baptiste Morin)	ボルドー大学・Department of Mathematics・CNRS Researcher	

7. 科研費を使用して開催した国際研究集会

〔国際研究集会〕 計1件

国際研究集会 Motives in Tokyo 2019	開催年 2019年～2019年
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8. 本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関		
フランス	ボルドー大学		