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研究課題名(和文) Index theorems in scattering theory: beyond a finite number of bound states

研究課題名(英文) Index theorems in scattering theory: beyond a finite number of bound states

研究代表者

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研究成果の概要(和文)：We developed analytical tools in the context of quantum scattering theory. They exhibit properties of physical systems which are robust under perturbations. We also performed bibliometric research on mathematical papers, and developed new tools for computing the effective reproduction number.

研究成果の学術的意義や社会的意義

Stability results of quantum systems are important, since these systems are constantly subject to perturbations. Bibliometric investigations provide a clear link between international collaborations and citations. New methods for computing the effective reproduction number can have a huge impact.

研究成果の概要(英文)：We have developed analytical tools in the context of quantum scattering theory. These results are necessary for exhibiting properties of physical systems which are robust under perturbations. Because of the pandemic, we have also performed bibliometric research on mathematical papers, and developed new tools for computing the effective reproduction number of the COVID-19 epidemic.

研究分野：Mathematics

キーワード：Scattering theory Wave operators Index theorems Integrable models Bibliometric analysis
Epidemiology

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様式 C-19、F-19-1、Z-19 (共通)

1. 研究開始当初の背景 Background of the research

In 1949, the American mathematician N. Levinson published a paper which contains in a parenthesis a rather surprising equality: the number of bound states of a physical system can be computed from the scattering part of the system. During more than 50 years, physicists and mathematicians have then extensively generalized this result (also called Levinson's theorem) and numerous research papers and chapters in books are devoted to this equality. However, it is only in 2006 that we mentioned for the first time that this equality is in fact an index theorem and that a C*-algebraic framework can tremendously extend the applicability and the interest of such an equality.

2. 研究の目的 Purpose of the research

Our approach takes its root in the algebraic framework of non-commutative geometry developed by A. Connes and in a very precise analysis of one of the main objects of scattering theory, namely the wave operators. For the last 15 years, our general aim has been to show how fruitful the interactions of these two subjects (non-commutative geometry and scattering theory) can be. For example, it led to the exhibition of new explicit formulas for the wave operators for many physical models, and prompted the development of new purely analytical tools, as some new formulas for the resolvent expansion at embedded thresholds. New propagation estimates are also necessary for the applicability of the algebraic framework. More recently, some links have also been exhibited between our topological approach of Levinson's theorem and very recent researches on topological insulators. These developments lead again to the need of refined analytical estimates for their implementation in an algebraic framework. Thus, our general aim is to develop some analytical tools and further develop the C*-algebraic framework such that these two subjects can be combined harmoniously and lead to new unexpected results.

3. 研究の方法 Research methods

Since this research was mainly based on the study of models of quantum systems we spent a lot of time performing spectral and scattering investigations on such models. These investigations were realized in collaboration with several researchers worldwide, and with my PhD students H. Inoue and Q. Sun. Thus, during the first two years of this proposal we visited several collaborators in Japan, in France, in Chile, and in China, and we also invited some of them for collaboration in Japan. During these two years, we also attended several conferences and delivered numerous talks for discussing the results of our investigations. Then, during the FY2020 and FY2021 the COVID-19 pandemic had a huge impact on our research program: no trips were possible, meaning that no visit to collaborators abroad took place, and no collaborators could be invited to Japan. After a period of transition, we have adapted our research methods to this new framework, and reoriented part of our research. As a consequence, the diversity of the results obtained has increased. Another consequence of the pandemic has been the extension of this research project, from 3 years to 4 years.

4. 研究成果 Research results

As indicated below, the outcomes of this period of research consists in 14 published papers. Most of these publications are directly linked to the initial project, with several developments on various aspects of spectral and scattering theory, and a few papers are related to the new projects developed because of the pandemic.

For the projects directly related to the initial proposal, let us mention [8,11,12,16], in which integrable models are thoroughly investigated, from the analytical point of view and then from the C^* -algebraic point of view. These papers contain new and unexpected extensions of Levinson's theorem, and [11] has been published in the leading journal of non-commutative geometry. The two companion papers [10] and [15] contain a fairly deep analysis of so called quantum walk systems in an anisotropic setting. The complementarity of the analytical approach and of the algebraic approach is the key of some of the proofs. Papers [3,4,5,7,9,14] contain additional investigations in the framework of spectral and scattering theory. Among them, [5] deals with a study of photonic crystals, while [7] contains complementary analytical and algebraic results on 2D Schrödinger. Let us finally mention [2] and [13] with contain propagation estimates obtained in a very general framework and with numerous applications.

For the projects not initially planned, let us mention [6] in which we have performed a bibliometric research on papers published in mathematics. This is a subject that we wanted to investigate for a long time, and the restrictions due to the pandemic gave us the perfect opportunity to develop it. Another project focusing directly on COVID-19 has resulted in the publication [1]. This paper corresponds to a fruitful collaboration with prof. Miyoshi, from RIKEN, and with an undergraduate student and a master student. None of us had a prior knowledge on epidemiology, but together we developed new tools for computing the effective reproduction number, one of the main indicators for the propagation of an epidemic. These investigations are based on a particle filter scheme, one technique used in data assimilation, the field of expertise of prof. Miyoshi.

Publications:

- 1) C. Sun, S. Richard, T. Miyoshi, N. Tsuzu, Analysis of COVID-19 spread in Tokyo through an agent-based model with data assimilation, *J. Clin. Med.* 11, 2401 (17 pages), 2022.
- 2) S. Richard, R. Tiedra de Aldecoa, Decay estimates for unitary representations with applications to continuous- and discrete-time models, to appear in *Ann. Henri Poincaré*, 2022.
- 3) S. Richard, N. Tsuzu, Spectral and scattering theory for topological crystals perturbed by infinitely many new edges, *Rev. Math. Phys.* 33, 2250010 (26 pages), 2022.
- 4) S. H. Nguyen, S. Richard, R. Tiedra de Aldecoa, Discrete Laplacian in a half-space with a periodic surface potential I: Resolvent expansions, scattering matrix, and wave operators, *Mathematische Nachrichten* 295, 912 – 949, 2022.
- 5) G. De Nittis, M. Moscolari, S. Richard, R. Tiedra de Aldecoa, Spectral and scattering theory of one-dimensional coupled photonic crystals, *Rev. Math. Phys.* 33, 2150027 (40 pages), 2021.
- 6) S. Richard, Q. Sun, Bibliometric analysis on mathematics, 3 snapshots: 2005, 2010, 2015, *Electron. J. App. Stat. Anal.* 14, 90 – 116, 2021.
- 7) S. Richard, R. Tiedra de Aldecoa, L. Zhang, Scattering operator and wave operators for 2D Schrodinger operators with threshold obstructions, *Complex Anal. Oper. Theory* 15, article 106 (25 pages), 2021.

- 8) J. Dereziński, J. Faupin, Q.N. Nguyen, S. Richard, On radial Schrödinger operators with a Coulomb potential: General boundary conditions, *Advances in Operator Theory* 5, 1132 – 1192, 2020.
- 9) S. Richard, T. Umeda, On some integral operators appearing in scattering theory, and their resolutions, in *Spectral Theory and Mathematical Physics*, 243 – 256, Latin American Mathematics Series, Springer, 2020.
- 10) S. Richard, A. Suzuki, R. Tiedra de Aldecoa, Quantum walks with an anisotropic coin II: scattering theory, *Lett. Math. Phys.* 109, 61 – 88, 2019.
- 11) H. Inoue, S. Richard, Index theorems for Fredholm, semi-Fredholm, and almost periodic operators: all in one example, *J. Noncommut. Geom.* 13, 1359 – 1380, 2019.
- 12) H. Inoue, S. Richard, Topological Levinson's theorem for inverse square potentials: complex, infinite, but not exceptional, *Rev. Roumaine Math. Pures Appl.* 64 (2&3), 225 – 250, 2019.
- 13) S. Richard, R. Tiedra de Aldecoa, Commutator criteria for strong mixing II: More general and simpler, *CUBO* 21, 37 – 48, 2019.
- 14) D. Parra, S. Richard, Spectral and scattering theory for Schrödinger operators on perturbed topological crystals, *Rev. Math. Phys.* 30, 1850009-1 – 1850009-39, 2018.
- 15) S. Richard, A. Suzuki, R. Tiedra de Aldecoa, Quantum walks with an anisotropic coin I: spectral theory, *Lett. Math. Phys.* 108, 331 – 357, 2018.
- 16) J. Dereziński, S. Richard, On radial Schrödinger operators with a Coulomb potential, *Ann. Henri Poincaré* 19, 2869 – 2917, 2018.

Presentations:

- 1) Global Noncommutative Geometry Seminar (Worldwide), online, December 6, 2021.
- 2) Conference **Himeji Conference on Partial Differential Equations** (JP), online, March 4 - 5, 2021.
- 3) Conference **Spectral and Scattering Theory and Related Topics**, Rims Kyoto (JP), December 2 - 4, 2019.
- 4) Mini-course, Spring School in Analysis and Mathematical Physics, Pontificia Universidad Católica de Chile, Santiago (Chile), Oct. 16 - 18, 2019.
- 5) Colloquium of mathematics, Universidad de Chile, Santiago (Chile), August 28, 2019.
- 6) Workshop on **Analysis, PDE and mathematical physics**, Central China Normal University, Wuhan (China), July 25 - 26, 2019.
- 7) Conference **Spectral theory and mathematical physics**, Santiago (Chile), December 3 - 7, 2018.
- 8) Seminar of geometry, University of Savoie (F), September 27, 2018.
- 9) Seminar geometry, algebra and operator algebras, University Clermont Auvergne (F), Sept. 21, 2018.
- 10) Workshop **Scaling invariant structures for spectral analysis and perturbation theory**, RIMS Kyoto (JP), September 10 - 12, 2018.
- 11) Kagurazaka analysis seminar, Tokyo University of Science (JP), July 28, 2018.
- 12) Workshop **Mathematical aspects of quantum fields and related topics**, RIMS Kyoto (JP), July 2 - 4, 2018.

5. 主な発表論文等

〔雑誌論文〕 計16件（うち査読付論文 16件 / うち国際共著 16件 / うちオープンアクセス 5件）

1. 著者名 S. Richard, N. Tsuzu	4. 巻 33
2. 論文標題 Spectral and scattering theory for topological crystals perturbed by infinitely many new edges	5. 発行年 2022年
3. 雑誌名 Reviews in Mathematical Physics	6. 最初と最後の頁 26 pages
掲載論文のDOI (デジタルオブジェクト識別子) 10.1142/S0129055X22500106	査読の有無 有
オープンアクセス オープンアクセスとしている (また、その予定である)	国際共著 該当する
1. 著者名 S. Richard, R. Tiedra de Aldecoa	4. 巻 in press
2. 論文標題 Decay estimates for unitary representations with applications to continuous- and discrete-time models	5. 発行年 2022年
3. 雑誌名 Annales Henri Poincare	6. 最初と最後の頁 27 pages app.
掲載論文のDOI (デジタルオブジェクト識別子) なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する
1. 著者名 C. Sun, S. Richard, T. Miyoshi, N. Tsuzu	4. 巻 11
2. 論文標題 Analysis of COVID-19 Spread in Tokyo through an Agent-Based Model with Data Assimilation	5. 発行年 2022年
3. 雑誌名 Journal of clinical medicine	6. 最初と最後の頁 17 pages
掲載論文のDOI (デジタルオブジェクト識別子) なし	査読の有無 有
オープンアクセス オープンアクセスとしている (また、その予定である)	国際共著 該当する
1. 著者名 S. Richard, Q. Sun	4. 巻 14
2. 論文標題 Bibliometric analysis on mathematics, 3 snapshots: 2005, 2010, 2015	5. 発行年 2021年
3. 雑誌名 Electronic Journal of Applied Statistical Analysis	6. 最初と最後の頁 90 - 116
掲載論文のDOI (デジタルオブジェクト識別子) なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1. 著者名 S. H. Nguyen, S. Richard, R. Tiedra de Aldecoa	4. 巻 295
2. 論文標題 Discrete Laplacian in a half-space with a periodic surface potential I: Resolvent expansions, scattering matrix, and wave operators	5. 発行年 2022年
3. 雑誌名 Mathematische Nachrichten	6. 最初と最後の頁 912 - 949
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オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1. 著者名 G. De Nittis, M. Moscolari, S. Richard, R. Tiedra de Aldecoa	4. 巻 33
2. 論文標題 Spectral and scattering theory of one-dimensional coupled photonic crystals	5. 発行年 2021年
3. 雑誌名 Reviews in Mathematical Physics	6. 最初と最後の頁 2250010
掲載論文のDOI (デジタルオブジェクト識別子) なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1. 著者名 J. Dereziński, J. Faupin, Q.N. Nguyen, S. Richard	4. 巻 5
2. 論文標題 On radial Schroedinger operators with a Coulomb potential: General boundary conditions	5. 発行年 2020年
3. 雑誌名 ADVANCES IN OPERATOR THEORY	6. 最初と最後の頁 1132 - 1192
掲載論文のDOI (デジタルオブジェクト識別子) なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1. 著者名 S. Richard, T. Umeda	4. 巻 in press
2. 論文標題 On some integral operators appearing in scattering theory, and their resolutions	5. 発行年 2020年
3. 雑誌名 book: Spectral Theory and Mathematical Physics, proceeding of a conference	6. 最初と最後の頁 243 - 256
掲載論文のDOI (デジタルオブジェクト識別子) なし	査読の有無 有
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2. 論文標題 Index theorems for Fredholm, semi-Fredholm, and almost periodic operators: all in one example	5. 発行年 2019年
3. 雑誌名 Journal of noncommutative geometry	6. 最初と最後の頁 1359 - 1380
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1. 著者名 H. Inoue, S. Richard	4. 巻 64
2. 論文標題 Topological Levinson's theorem for inverse square potentials: complex, infinite, but not exceptional	5. 発行年 2019年
3. 雑誌名 Revue roumaine de mathematiques pures et appliquees	6. 最初と最後の頁 225 - 250
掲載論文のDOI (デジタルオブジェクト識別子) なし	査読の有無 有
オープンアクセス オープンアクセスとしている (また、その予定である)	国際共著 該当する

1. 著者名 S. Richard, R. Tiedra de Aldecoa, L. Zhang	4. 巻 15
2. 論文標題 Scattering Operator and Wave Operators for 2D Schroedinger Operators with Threshold Obstructions	5. 発行年 2021年
3. 雑誌名 Complex Analysis and Operator Theory	6. 最初と最後の頁 25 pages
掲載論文のDOI (デジタルオブジェクト識別子) 10.1007/s11785-021-01153-z	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1. 著者名 S. Richard A. Suzuki, R. Tiedra de Aldecoa	4. 巻 108
2. 論文標題 Quantum walks with an anisotropic coin I: spectral theory	5. 発行年 2018年
3. 雑誌名 Letters in Mathematical Physics	6. 最初と最後の頁 331 - 357
掲載論文のDOI (デジタルオブジェクト識別子) 10.1007/s11005-017-1008-1	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1. 著者名 S. Richard, A. Suzuki, R. Tiedra de Aldecoa	4. 巻 109
2. 論文標題 Quantum walks with an anisotropic coin II: scattering theory	5. 発行年 2018年
3. 雑誌名 Letters in Mathematical Physics	6. 最初と最後の頁 61 - 88
掲載論文のDOI (デジタルオブジェクト識別子) 10.1007/s11005-018-1100-1	査読の有無 有
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1. 著者名 J. Dereziński, S. Richard	4. 巻 19
2. 論文標題 On Radial Schroedinger Operators with a Coulomb Potential	5. 発行年 2018年
3. 雑誌名 Annales Henri Poincare	6. 最初と最後の頁 2869 - 2917
掲載論文のDOI (デジタルオブジェクト識別子) 10.1007/s00023-018-0701-7	査読の有無 有
オープンアクセス オープンアクセスとしている (また、その予定である)	国際共著 該当する

1. 著者名 S. Richard, R. Tiedra de Aldecoa	4. 巻 21
2. 論文標題 Commutator criteria for strong mixing II. More general and simpler	5. 発行年 2019年
3. 雑誌名 Cubo (Temuco)	6. 最初と最後の頁 37 - 48
掲載論文のDOI (デジタルオブジェクト識別子) 10.4067/S0719-06462019000100037	査読の有無 有
オープンアクセス オープンアクセスとしている (また、その予定である)	国際共著 該当する

1. 著者名 D. Parra, S. Richard	4. 巻 30
2. 論文標題 Spectral and scattering theory for Schroedinger operators on perturbed topological crystals	5. 発行年 2018年
3. 雑誌名 Reviews in Mathematical Physics	6. 最初と最後の頁 39 pages
掲載論文のDOI (デジタルオブジェクト識別子) 10.1142/S0129055X18500095	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

[学会発表] 計12件(うち招待講演 12件/うち国際学会 0件)

1. 発表者名 S. Richard
2. 発表標題 Bibliometric analysis on mathematics 3 snapshots: 2005, 2010, 2015
3. 学会等名 Himeji Conference on Partial Differential Equations (招待講演)
4. 発表年 2021年

1. 発表者名 S. Richard
2. 発表標題 Discrete Laplacian in a half-space with a periodic surface potential: - Resolvent expansions, scattering matrix, and wave operators
3. 学会等名 Spectral and Scattering Theory and Related Topics, Rims Kyoto (招待講演)
4. 発表年 2019年

1. 発表者名 S. Richard
2. 発表標題 Operator algebras: what are they good for ?
3. 学会等名 Spring School in Analysis and Mathematical Physics, Pontificia Universidad Catolica de Chile, Santiago, Chile (招待講演)
4. 発表年 2019年

1. 発表者名 S. Richard
2. 発表標題 Levinson's theorem in scattering theory: classical, topological, complex, infinite !
3. 学会等名 Workshop on Analysis, PDE, and Mathematical physics, Central China Normal University, Wuhan (China) (招待講演)
4. 発表年 2019年

1 . 発表者名 S. Richard
2 . 発表標題 Index theorems for Fredholm, semi-Fredholm and almost-periodic operators; all in one example
3 . 学会等名 Colloquium of mathematics, Universidad de Chile, Santiago (Chile) (招待講演)
4 . 発表年 2019年

1 . 発表者名 S. Richard
2 . 発表標題 Index theorems for Fredholm, semi-Fredholm and almost-periodic operators; all in one example
3 . 学会等名 International Conference Spectral Theory and Mathematical Physics, Santiago (Chile) (招待講演)
4 . 発表年 2018年

1 . 発表者名 S. Richard
2 . 発表標題 Le theoreme de Levinson topologique: 13 ans apres
3 . 学会等名 Seminar of geometry, algebra and operator algebras, Clermont-Ferrand University (F) (招待講演)
4 . 発表年 2018年

1 . 発表者名 S. Richard
2 . 発表標題 Wave operators: Beyond existence, completeness, and boundedness
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4 . 発表年 2018年

1. 発表者名 S. Richard
2. 発表標題 Theorie spectrale et de la diffusion sur des cristaux topologiques perturbés
3. 学会等名 Seminar geometry, algebra and operator algebras, University Clermont Auvergne (F) (招待講演)
4. 発表年 2018年

1. 発表者名 S. Richard
2. 発表標題 Spectral and scattering theory for Schroedinger operators on perturbed topological crystals
3. 学会等名 Workshop Mathematical aspects of quantum fields and related topics, RIMS Kyoto (JP) (招待講演)
4. 発表年 2018年

1. 発表者名 S. Richard
2. 発表標題 Schroedinger operators on the half-line with Coulomb and centrifugal terms: spectral and scattering results
3. 学会等名 Workshop Scaling invariant structures for spectral analysis and perturbation theory, RIMS Kyoto (JP) (招待講演)
4. 発表年 2018年

1. 発表者名 S. Richard
2. 発表標題 Scattering theory and non-commutative geometry: A walk from the parentheses of Levinson to the hexagon of Cordes
3. 学会等名 Global Noncommutative Geometry Seminar (Worldwide), online (招待講演)
4. 発表年 2021年

〔図書〕 計0件

〔産業財産権〕

〔その他〕

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6. 研究組織

	氏名 (ローマ字氏名) (研究者番号)	所属研究機関・部局・職 (機関番号)	備考
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7. 科研費を使用して開催した国際研究集会

〔国際研究集会〕 計3件

国際研究集会 Himeji conference on partial differential equations	開催年 2019年～2019年
国際研究集会 Himeji conference on partial differential equations	開催年 2021年～2021年
国際研究集会 Himeji conference on partial differential equations	開催年 2022年～2022年

8. 本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関
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