【Grant-in-Aid for Scientific Research (S)】

Broad Section B

Title of Project : Creation of advanced method in mathematical analysis on nonlinear mathematical models of critical type

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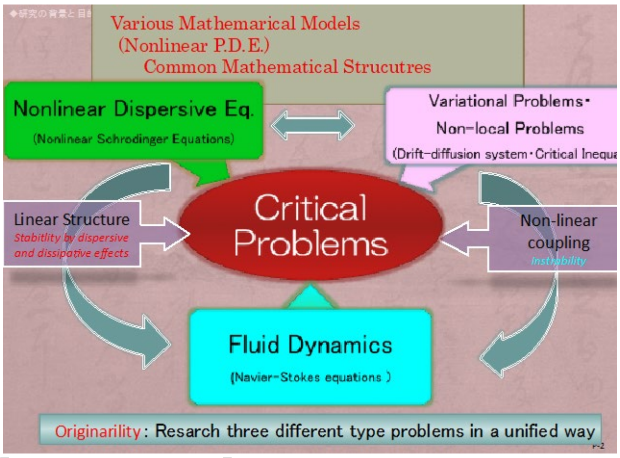
Research Project Number : 19H05597 Researcher Number : 20224017

Keyword : Nonlinear Partial Differential Equations, Variational Method, Critical Inequalities, Harmonic Analysis

【Purpose and Background of the Research】

Many of mathematical models are described by nonlinear partial differential equations and such equations typically have both linear and nonlinear structures therein. The linear part is described by partial differential operators by local space-time variables and the nonlinear part is produced by the interaction between different physical quantities and the linear structure stabilize the system while the nonlinear part causes instability of the model. Between those effects, there exists a sort of problems where the both effects are analytically balanced. We call this type of problem as the "Critical Problems" and it is our main subject of this project. Problems of this type are interesting both from an applied and a theoretical mathematical point of view. They often lead to new and fascinating open problems. A serious difficulty in the study of such "critical problems", is that the analysis derived through perturbation theory is not directly applicable and a new methodology has to be developed.



【Research Methods】

The critical problems are in general equipped by standard and natural structures corresponding to conservation laws, for the conservation of mass, momentum and energy. These standard structures are given by entropy dissipation, Galilei invariance as well as by conformal invariance. These structures are essential in the study of the critical problems. One of the main tools used for this research are variational methods. Also fundamental for our study is the possibility of developing new functional inequalities of critical type. In particular with the aid of functional and harmonic analysis tools such as real interpolation methods, we develop a new critical inequalities such as Trudinger-Moser type involving Shannon-Renyi Entropy functionals and develops the linear estimates for dispersive space time estimates and end-point maximal regularity for the dissipative system which will be then used to better understand the ``critical problems" and even beyond the critical problem.

【Expected Research Achievements and Scientific Significance】

Important unsolved problems, such as the well known millennium ones, which describe central issues in mathematics are still wide open. An important challenge is to find unified methods to overcome the difficulties that stem from the interaction of both dissipative and dispersive behaviors. We believe that a finer analysis on the nonlinear structure make us possible to treat those unsolved problems after establishing a new type of critical estimates for linear and nonlinear structures including dissipative and dispersive estimate such as Strichartz estimate and maximal regularity estimates.

【Publications Relevant to the Project】

「Real Analytic Method for Nonlinear Evolutional Partial Differential Equations」 Springer-Verlag 570pp, 2019, to appear.

【Term of Project】 FY 2019-2023

【Budget Allocation】 100,900 Thousand Yen

【Homepage Address and Other Contact Information】

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