【Grant-in-Aid for Scientific Research (S)】

**Broad Section B**

# Title of Project： Systematical geometric analysis and asymptotic analysis for evolution equations

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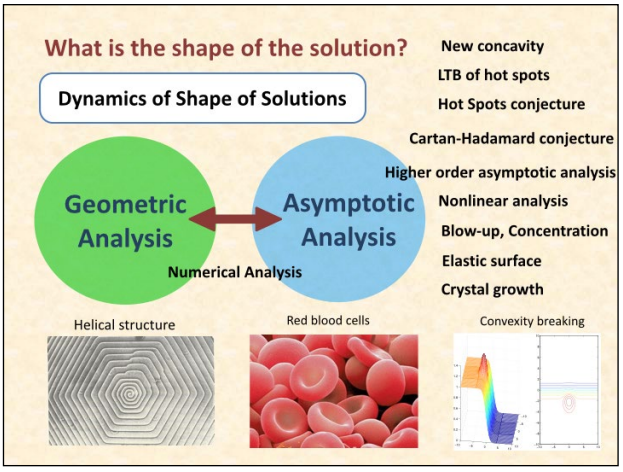
Research Project Number：19H05599　Researcher Number：90272020
Keyword：evolution equation, geometric analysis, asymptotic analysis, power concavity, blow-up problems

## 【Purpose and Background of the Research】

Many of the mathematical models appearing in mathematical sciences such as physics, chemistry, biology and astronomy are described by use of solutions of partial differential equations. Geometric analysis and asymptotic analysis give fruitful information in the study of the mathematical models. In particular, it is a natural intellectual desire to know the behavior of the solutions and their geometric properties. Furthermore, geometric analysis and asymptotic analysis are strong methods for the study of nonlinear phenomena related to blow-up and aggregation. In this research project we study the behavior of solutions of evolution equations and their system by use of geometric analysis and asymptotic analysis and investigate qualitative properties of the solutions. Furthermore, we try to find new interesting problems and the directions of their related fields.

## 【Research Methods】

This research project concerns with the simple problem "what is the shape of solutions of evolution equations and their systems?" We study geometric properties of the solutions and try to understand the mechanism of the change of the shape of the solutions. This project is based on the geometric analysis and the asymptotic analysis. Firstly, we study power concavity of solutions of elliptic equations and evolution equations and their systems and try to find a new notion of concavity. Our analysis is based on viscosity solutions and we expect that it is applicable to various nonlinear evolution equations. Furthermore, developing the asymptotic analysis, we study the behavior of landmark points such as hot spots and critical points of the solutions. As applications of our studies, we treats various nonlinear phenomena such as blow-up problems, aggregation of diffusion substances, convexity breakings, higher-order asymptotic analysis of solutions of evolution equations including higher-order parabolic equations and their systems, the structure of solutions of nonlinear elliptic equations, the movement of elastic surfaces and crystals, and the solutions with dynamical boundary conditions.




We hire several postdoc students and research assistants for our research project. We also organize international workshops related to geometric properties of solutions of evolution equations and blow-up problems.

## 【Expected Research Achievements and Scientific Significance】

The Research organization has unique viewpoints and research methods for geometric analysis and asymptotic analysis. Combining recent progresses in mathematical sciences, we can expect to obtain creative research results. We think that the results in this project greatly contribute to the development of the entire analysis since the project is related to fundamental inequalities in analysis and various fields in nonlinear phenomena.

## 【Publications Relevant to the Project】

F. Gazzola, K. Ishige, C. Nitsch, P. Salani eds., "Geometric Properties for Parabolic and Elliptic PDE's", Springer Proceedings in Mathematics & Statistics, Vol. 176, Springer International Publishing Switzerland (2016).

K. Ishige and P. Salani, Parabolic power concavity and parabolic boundary value problems, Math. Ann. 358 (2014), 1091−1117.

## 【Term of Project】

FY2019-2023

## 【Budget Allocation】

107,500 Thousand Yen

## 【Homepage Address and Other Contact Information】

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