様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９　（共通）

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研究課題名（和文）Tilting theory of gentle algebras via surface combinatorics

研究課題名（英文）Tilting theory of gentle algebras via surface combinatorics

研究代表者
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研究成果の概要（和文）：研究期間中、私は4つの研究プロジェクトを出版し、また4つのプレプリントを完成させ、査読のために提出しました。これらの研究プロジェクトのうち3つは提案された研究テーマと直接関連しています。具体的には、1つの論文は特定の環論的構成を介したBrauer木代数の拡大について、1つの論文は曲面トポロジーとの関連に基づいてgentle代数の捩じれ類を分類しています。さらに、1つの論文は設定から向き付き不可能な曲面な設定への代数と曲面トポロジーの関係を拡張しています。他の研究プロジェクトでは、有限次元代数のさまざまなホモロジー的性質を理解し、長年の問題にいくつかの突破口を提供しています。

研究成果の学術的意義や社会的意義
The research on classifying torsion classes provide a breakthrough in understanding this type of problems. The research on non-orientable surface also bring new connection between new area of representation theory and topology.

研究成果の概要（英文）：Over the period of the research, I have had four research projects published, as well as having four preprints finished and submitted for review. Three of these research projects are directly related to the proposed research theme. Namely, one article looks at enlargement of Brauer tree algebras via certain ring theoretical construction; one article classifies the torsion classes of gentle algebras, based on its connection with surface topology; and one article extending the connection between algebras and surface topology from the orientable setting to the non-orientable one. My other research projects revolve around understanding various homological properties of finite-dimensional algebras, providing various breakthrough in long standing questions. On top of these, I have also organised school on differential graded algebras and also school on Koszul algebras, bring together researchers across various areas.

研究分野：Algebra

キーワード：gentle algebra　tilting theory　surface topology　Brauer graph algebras　Calabi-Yau algebra　Auslander algebra

１．研究開始当初の背景

The origin of *gentle algebras* can be traced back to the study of representations over the Lorentz group by Gelfand-Ponomarev; it has been discovered recently that they are closely related to topological Fukaya categories of surfaces, categorification of cluster algebras, and modular representation theory of finite groups. We propose to study the derived categories of gentle algebras by looking at their tilting theory. More precisely, as an extension of the author's previous works on classification of two-term silting complexes, we are interested on a more relaxed gadget called *torsion classes* - full subcategories that are closed under quotients and extensions - of the module categories.

The notion of *torsion class*, i.e. full subcategories that are closed under quotients and extensions, in an abelian category was introduced in the dawn of using categorical technique to study representation theory. A particular example comes from the class of torsion groups in the category of abelian groups. It gave birth to *tilting theory* - in particular constructions of equivalences between derived categories - which is a central theme in modern representation theory. This research project aims to explore new techniques that can shed light on understanding torsion classes as well as tilting theory in general.

２．研究の目的

Our main aim is to classify these torsion classes using combinatorial tools coming from both algebra and geometry. Namely, by associating a gentle algebra with a ribbon graph embedded in a surface, we will establish a correspondence between certain curves on the surface and the indecomposable modules, and use this to classify torsion classes using the notion of laminations of surfaces. Moreover, we will use this result to classify certain silting complexes, and to determine the Bridgeland's stability conditions that are induced by torsion classes.

３．研究の方法

Our idea for classifying torsion classes of gentle algebras is to use its topological interpretation. Recall that ribbon graphs are planar graphs on (orientable) surfaces. It turns out that a gentle algebra Λ can be defined using a *ribbon surface* (S, Γ), which encodes how a ribbon graph Γ embeds into a surface S. Construct a quiver Q whose vertices correspond to the arcs of Γ. For each vertex of Γ, the orientation of S induces an ordering of arcs attached to each vertex, then arrows of Q correspond to pairs of consecutive arcs in these ordering. Then Λ associated to (S, Γ) is given by the path algebra of Q with relations generated by all compositions of arrows that do not appear as consecutive pair in these orderings.

This connection to topology allows one to regard Λ-modules and complexes as certain curves on the associated ribbon surface. Thus, one might expect some technology from topology of surfaces could bring new inspiration into classification of torsion classes for gentle algebras. In fact, using our previous works on a similar class of algebras, we already had a candidate object in surface topology that can be used to classify torsion classes, namely, the *laminations*.

Our first goal is to establish a correspondence between torsion classes of Λ and (generalised) laminations of the associated ribbon surface. An additional goal is to distinguish the *functorially finite* torsion classes from general torsion classes using surface topology. This helps to see the discrepancy between the study of tilting theory via silting complexes and via torsion classes.

４．研究成果

The research project has been successful. A preprint is written up and is currently under review. Namely, we established the correspondence between torsion classes of gentle algebras with a certain combinatorial tool called maximal noncrossing sets of strings. This is the algebro-combinatorial analogue of curves on surfaces. This work will provide the first step into further comparison between laminations in classical surface topology

- and those use in the algebraic study of tilting theory for gentle algebra.

Beside its topological significance, our work provides a breakthrough in the classification problem of torsion classes of finite-dimensional. Explicit classification were only known to very few cases before and showing only limited phenomenon, and we have now extended to a much larger classes where previously unseen phenomenon occur.

On top of this work, I have also written up works that are closely related to the proposed research projects. These concern generalisation of the aforementioned connection between tilting theory and surface topology. The most recent preprint in this direction give an attempt to generalise the setting of orientable surface topology to non-orientable ones. It tuns out that this require some rather new tools in representation theory. This work brings new potential breakthrough not only in surface topology, but also in new aspect of representation theory.

５．主な発表論文等

〔雑誌論文〕　計4件（うち査読付論文　4件／うち国際共著　3件／うちオープンアクセス　1件）

１．著者名 Takuma Aihara, Aaron Chan, Takahiro Honma	４．巻 603
２．論文標題 On representation-finite gendo-symmetric algebras with only one non-injective projective module	５．発行年 2022年
３．雑誌名 Journal of algebra	６．最初と最後の頁 61, 88
掲載論文のDOI（デジタルオブジェクト識別子） 10.1016/j.jalgebra.2022.04.002	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 Chan Aaron、Liu Yuming、Zhang Zhen	４．巻 560
２．論文標題 On simple-minded systems and τ-periodic modules of self-injective algebras	５．発行年 2020年
３．雑誌名 Journal of Algebra	６．最初と最後の頁 416～441
掲載論文のDOI（デジタルオブジェクト識別子） 10.1016/j.jalgebra.2020.05.024	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 Chan Aaron、Wong William	４．巻 4
２．論文標題 Irreducible representations of the symmetric groups from slash homologies of p-complexes	５．発行年 2021年
３．雑誌名 Algebraic Combinatorics	６．最初と最後の頁 125～144
掲載論文のDOI（デジタルオブジェクト識別子） 10.5802/alco.153	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

１．著者名 CHAN AARON、MIEMIETZ VANESSA	４．巻 243
２．論文標題 COIDEMPOTENT SUBCOALGEBRAS AND SHORT EXACT SEQUENCES OF FINITARY 2-REPRESENTATIONS	５．発行年 2020年
３．雑誌名 Nagoya Mathematical Journal	６．最初と最後の頁 316～341
掲載論文のDOI（デジタルオブジェクト識別子） 10.1017/nmj.2020.1	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

〔学会発表〕　計6件（うち招待講演　4件／うち国際学会　1件）

1．発表者名
Aaron Chan

2．発表標題
Categorification of quasi-triangulations of unpunctured non-orientable marked surfaces

3．学会等名
Geometric and homological methods in representation theory, Lancaster University（招待講演）（国際学会）

4．発表年
2022年

1．発表者名
Aaron Chan

2．発表標題
Categorification of quasi-triangulations of unpunctured non-orientable marked surfaces

3．学会等名
Infinte Analysis 21 Workshop Around Cluster Algebras（招待講演）

4．発表年
2021年

1．発表者名
Aaron Chan

2．発表標題
Recollement of comodule categories over coalgebra object

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The 8th China-Japan-Korea International Symposium on Ring Theory

4．発表年
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Aaron Chan

2．発表標題
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3．学会等名
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4．発表年
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Aaron

２．発表標題
Torsion classes of gentle algebras

３．学会等名
Oberwolfach meeting on Representation Theory of Quivers and Finite Dimensional Algebras

４．発表年
2020年

１．発表者名
Aaron Chan

２．発表標題
Tilting theory for algebras associated to surface dissections

３．学会等名
Mathematical Society of Japan 2022 Autumn meeting（招待講演）

４．発表年
2022年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

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６．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考

７．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計0件

８．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関