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研究課題名（和文）Probing local gravitational physics with Lorentzian singularities in field theory

研究課題名（英文）Probing local gravitational physics with Lorentzian singularities in field

研究代表者
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研究成果の概要（和文）：本研究の目的は有限温度共形場理論の二点関数に発生しうる特異点を研究することである．私は大栗博司教授とともに，有限温度共形場理論の二点関数の計算をブラックホール背景での揺らぎの計算にホログラフィー対応を用いて帰着することで実行した．計算の結果，ブラックホールを周回する光の軌道に対応する光円錐のため，存在しえない特異点が発生するという矛盾を発見した．また，この矛盾を弦理論を用いたより詳しい解析によって解決した．具体的には，ペンローズ極限と呼ばれるブラックホール時空の解析に適した極限を用いて，弦理論が正しく偽の特異点を消すことを示した．

研究成果の学術的意義や社会的意義
共形場理論は理論物理に普遍的に現れる重要な構造であり，実験的にはイジング模型の相転移点など二次相転移付近の物理に現れることが知られている．共形場理論を特徴づける量は相関関数であり，この構造を理解することが重要である．一方，本研究まではどのような特異点が二点関数に起こりえるかという疑問に精密な回答がなかったが，我々はこれまで知られていた特異点以外の特異点が発生しないことを示した．この研究によって，共形場理論の構造への理解が深まった．また，実験による検証可能性による新たな展開も存在する．共形場理論は理論物理以外の分野にも応用があるため，我々の結果はより広い文脈でも重要な可能性がある．

研究成果の概要（英文）：The goal of this research project was to understand the possible singularities of the thermal two-point function in conformal field theories. A conformal field theory is a special kind of quantum field theory with no scale. The two-point function measures the correlation between two points in spacetime, and is singular when the two points can be connected by a light ray. Using the idea of holography, the thermal two-point function can be computed by considering fluctuations in a black hole background. Together with Hirosi Ooguri, we showed that there are naively new singularities in the two-point function due to light rays in the black hole background. These light rays can wrap around the black hole many times. However, one needs to take string theory into account to analyze this problem in full detail. By analysing the so-called Penrose limit of the spacetime, we were able to compute the full string-theoretic answer, and showed that the singularities are resolved in string theory.

研究分野：String theory

キーワード：String theory　black holes　conformal field theory　AdS/CFT　二点関数

１．研究開始当初の背景

The AdS/CFT correspondence is a powerful tool for studying quantum field theories at strong coupling. In this correspondence, ordinary quantum field theories are related to theories with gravity in Anti de Sitter space. The theory of gravity in the bulk is weakly coupled, while the boundary theory is strongly coupled. Many questions in quantum field theory that cannot be solved using standard techniques are very simple in AdS/CFT, and this provides an avenue for understanding strongly coupled conformal field theories.

A special case of AdS/CFT is when the boundary theory is at finite temperature. In the bulk, the dual system is a black hole. Schwarzschild black holes in Anti de Sitter space reproduce many of the important features of the boundary conformal field theory, such as the thermal phase transition at a certain temperature. One important observable at finite temperature is the correlation function, which measures the correlation between two points in spacetime. In AdS/CFT, the thermal correlation functions can be computed by studying the wave equation on a black hole background.

In studying correlation functions of conformal field theories, one of the most fundamental questions is to understand the singularity structure. Euclidean correlation functions only have a singularity when two points approach each other, but when one studies real time processes the singularities can become more complicated. For example, there is a singularity in real time correlation functions whenever two points are connected by a light ray. At zero temperature, one can study the most general singularity using the AdS/CFT correspondence. One finds new singularities (the so-called bulk-point singularities) in addition to the standard light cone singularity. These new singularities arise from null geodesics in the bulk.

In the AdS/CFT correspondence, one should not just study gravity in the bulk, but a full theory of strings. A string theory is a theory with excitations that are not point like, but are instead extended in spacetime. It turns out that the bulk point singularities are resolved when one takes the full bulk string theory into account, even though they are naively present in the point particle approximation. In a sense, the bulk string theory smooths out the singularities, and makes the theory more well-behaved.

２．研究の目的

The purpose of this project was to understand the structure of singularities in correlation functions at finite temperature. At zero temperature, the bulk singularities are present in four-point functions, but at finite temperature one can look at a simpler object, the two-point function. The two-point function contains many of the interesting finite temperature physics, such as hydrodynamics and quasi-normal modes. Therefore we can ask where are the possible real time singularities of the thermal two-point function.

There is one kind of singularity that must be present - the light cone singularity, where the two points are connected by a boundary light ray. A more nontrivial type of singularity comes from a bulk light ray that extends into the black hole background. Just like the bulk point singularity at zero temperature, we expect naive singularities whenever two points on the boundary are connected by a geodesic in the bulk. We can then try to understand whether these naive singularities are resolved by string theory, or if they are real singularities. This is an important question, since we would like to understand what consistency conditions correlation functions must satisfy in conformal field theory at finite temperature.

３．研究の方法

In order to study the problem described above, we first had to solve for null geodesics in the black hole background. Since there are two conserved quantities (the energy and angular momentum), the geodesic equation can be integrated to find the explicit form of null geodesics. One then needs to solve for the null geodesic connecting two boundary points. This problem simplifies in two limits. The first limit (the early-time limit) is when the geodesic is far away from the black hole. In this case the geodesics can be approximated by geodesics in Anti de Sitter space. The second limit (the late-time limit) is when the two points on the boundary are separated by a long time separation. In this case the bulk null geodesic approaches the so-called photon sphere of the black hole. The bulk geodesic can then wrap the photon sphere many times. We showed how the geodesics simplify in both of these limits.

After computing the null geodesics, one has to compute the correlation function using the bulk quantum field theory. A useful approximation is to take the limit where the mass of the bulk field is large. On the boundary, this corresponds to considering large conformal dimensions. In this limit, the correlation function can be computed by considering space like geodesics that are close to the given null geodesic. The correlation function is determined by the length of these space like geodesics. We showed how to compute the correlation function in both the early time and late time limit, and showed that it is singular when the two points are null separated.

After considering the correlation function in the bulk quantum field theory, we considered the full string theory in order to understand whether the singularity was resolved. String theory in a black hole background is very complicated. In particular, there is no known solution to the nonlinear sigma model with a black hole target space. Therefore we needed to consider an approximation to the geometry known as the Penrose limit. The Penrose limit describes the behavior of a geometry near a null geodesic. Fortunately, it is known how to solve the bulk string theory in the Penrose limit. The theory reduces to a collection of harmonic oscillators with time dependent frequencies. We considered these harmonic oscillators using three different approximations - the Born approximation, the shockwave approximation, and the WKB approximation.

The Born approximation and the shockwave approximation can be combined to obtain a calculation of the string theoretic two-point function at early times. Meanwhile the WKB approximation can be used to obtain the answer at late times. The answer then takes the form of the point particle answer, multiplied by a product of determinants of infinitely many time-dependent harmonic oscillators. These determinants can be computed using the Gelfand-Yaglom theorem, which relates the determinants to solutions of a certain differential equation. Using these methods we computed the full two point function in momentum space, and showed that the string theoretic corrections lead to exponential decay at large momentum.

４．研究成果

The result of this project was that even though there are naively new singularities in the two-point function at finite temperature, they are resolved by string theory in the bulk. This is analogous to the case of the four-point function at zero temperature, where the bulk-point singularities are smoothed out by string theory. This leads to a new confirmation of the importance of string theory for understanding the structure of strongly coupled quantum field theories. In addition, our work provides new insight into the general structure of conformal field theory at finite temperature, which is an area of research with much potential for new investigation.

I presented this work at many international institutions, such as Berkeley and CERN. In addition, my coauthor Professor Ooguri Hirosi gave talks on our paper at Stanford and UC Santa Barbara. This work led to a new collaboration with Professor Alexander Zhiboedov at CERN, which I will now briefly describe.

One interesting aspect of the thermal two-point function is its poles. These are known as quasi normal modes, which describe resonances in the wave equation. The quasi normal modes describe the eventual thermalization of approximate energy eigenstates. In our work with Zhiboedov, we studied the boundary dual of the quasi normal modes. In the limit of large spin, these resonances become exact eigenstates, which therefore describe operators with fixed energy on the boundary. We showed that these operators are the so called heavy-light double twist operators, which have been investigated in great detail recently in the context of the conformal bootstrap. We determined exact formulas for the dimensions of these quasi normal modes in a certain semiclassical limit, and showed that these formulas reduce to the known results from conformal bootstrap in perturbation theory. In addition, we studied the thermalization of the approximate energy eigenstates, which occurs via gravitational radiation and tunneling. This work also connects to the concept of quantum many-body scars from condensed matter physics. A quantum scar is an energy eigenstate which are not thermal. In our context, the heavy-light double twist operators are exact energy eigenstates perturbatively at large spin, which have non-thermal behavior. However, it turns out that they thermalize if one considers nonperturbative corrections. Therefore these operators are approximate scars, which leads to a new connection between AdS/CFT and condensed matter physics.

I am continuing to work on related topics, since I believe there are many interesting directions to explore in this direction. One interesting work that will be completed shortly is in collaboration with Alba Grassi, Cristoforo Iossa, Daniel Lichtig, and Alexander Zhiboedov. Until now, there is no known exact solution for the thermal two-point function in the literature. We are studying this problem by analyzing the exact solutions to the wave equation in the bulk, which is known as the Heun equation. The Heun equation can be analyzed exactly using techniques from supersymmetric quantum field theory. Using these techniques, we were able to compute the exact two point function. Interestingly, this leads to an exact solution for the quasi normal modes, which matches the known results when one considers perturbation theory in the mass of the black hole. We were also able to compute the residues of the thermal two point function at the poles, which correspond to three point functions on the boundary. This work has many potential applications to finite temperature physics, including hydrodynamics and the study of the black hole singularity.

５．主な発表論文等

〔雑誌論文〕　計2件（うち査読付論文　2件／うち国際共著　2件／うちオープンアクセス　0件）

１．著者名 Ashwinkumar Meer、Dodelson Matthew、Kidambi Abhiram、Leedom Jacob M.、Yamazaki Masahito	４．巻 2021
２．論文標題 Chern-Simons invariants from ensemble averages	５．発行年 2021年
３．雑誌名 Journal of High Energy Physics	６．最初と最後の頁 None
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/JHEP08(2021)044	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 Dodelson Matthew、Ooguri Hirosi	４．巻 103
２．論文標題 Singularities of thermal correlators at strong coupling	５．発行年 2021年
３．雑誌名 Physical Review D	６．最初と最後の頁 1-14
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevD.103.066018	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

〔学会発表〕　計1件（うち招待講演　1件／うち国際学会　0件）

１．発表者名 Matthew Dodelson
２．発表標題 Chern-Simons Invariants from Ensemble Averages
３．学会等名 Number Theory, Strings, and Quantum Physics（招待講演）
４．発表年 2021年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

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６．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考

７．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計0件

８．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関			
米国	California Institute for Technology	University of California, Berkeley		
スイス	CERN			