様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９　（共通）

科学研究費助成事業　　研究成果報告書

令和　５ 年　 ６ 月 ２９ 日現在

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研究課題名（和文）Cyclotomic KLR algebras in type C: cellularity and blocks

研究課題名（英文）Cyclotomic KLR algebras in type C: cellularity and blocks

研究代表者
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交付決定額（研究期間全体）：（直接経費）　　2,200,000円

研究成果の概要（和文）：プレプリントにおいてA 型岩堀ヘッケ代数のブロックのシューア有限性（タウ傾有限性と言ってもよい）を決定した。その結果、ブロックが有限表現型を持つ場合に限りシューア有限であることがわかった。
また、 C型 円分 KLR 代数 R\Lambda_n の次数付分解係数を計算するための 2 つのアルゴリズムを開発し\Lambdaがレベル 1 かつn<13の場合に次数付分解行列をすべて計算した。この同じプロジェクトで標数 0 の 体上定義されたSpecht 加群の部分加群構造をn<11 の場合に計算し、A型と異なり対応する標準基底から得られる係数が標数 0 の次数付分解係数に一致しない最初の例を得た。

研究成果の学術的意義や社会的意義
Schurian-finiteness is a property which many researchers in finite-dimensional algebras seek to determine for algebras.
The KLR algebras arose from categorification of quantum groups and are studied a lot recently as part of a broader program of categorification. Many open questions remain.

研究成果の概要（英文）：We determined the Schurian-finiteness, or equivalently the tau-tilting finiteness, of blocks of type A Iwahori-Hecke algebras, in a preprint (arXiv:2112.11148) submitted for publication. Our main result is that blocks are Schurian-finite if and only if they have finite representation type (known to be the case if and only if they have weight 0 or 1). This project made use of a great breadth of tools, both existing and newly developed for our work.
We've also developed 2 algorithms for computing graded decomposition numbers for cyclotomic KLR algebras R\Lambda_n in type C, and computed all such graded decomposition matrices in level 1, for n <13. In this same project, we also computed the submodule structure of Specht modules in characteristic 0 for n<11, and obtained the first example of characteristic 0 graded decomposition numbers that are not given by the corresponding canonical basis coefficients. The paper is being written up, and we aim to have a preprint submitted by the autumn.

研究分野：Representation theory

キーワード：Cyclotomic KLR algebras　Hecke algebras　Schurian-finiteness　Specht modules

様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９（共通）

１．研究開始当初の背景

In 2008, Khovanov–Lauda, and independently Rouquier, introduced some algebras known as quiver Hecke algebras or KLR algebras, in order to categorify the negative half of quantum groups, along with their cyclotomic quotients. Shortly after, Kang–Kashiwara proved that the cyclotomic quotients categorify highest weight irreducible modules over the corresponding quantum groups.
The cyclotomic KLR algebras have been especially well-studied in type A, in no small part thanks to the Brundan–Kleshchev isomorphism, connecting them to cyclotomic Hecke algebras that had previously been studied in more depth. In other types, the cyclotomic quotients remain much more mysterious. However, in (affine or infinite) type C, we now have a combinatorial framework for studying these algebras, analogous to that in type A. We have a family of Specht modules indexed by multipartitions, with bases indexed by standard tableaux, etc. These were introduced previously by the principal investigator, in collaboration with Ariki and Park. Thus we have the beginnings of the systematic study of the representation theory of type C cyclotomic KLR algebras.
This research project aims to develop the representation theory of these type C algebras further.

２．研究の目的

In our attempts to develop the representation theory of cyclotomic KLR algebras in type C, we had originally planned to prove that these algebras were cellular, proving along the way that the Ariki–Park–Speyer Specht modules are the cell modules, and that they have bases indexed by standard tableaux; this was previously only shown in some cases. However, during the course of the project, Evseev–Mathas released a proof of these results using a clever deformation construction. Thus we shifted our attention to what developments can be made with these new tools, such as working on decomposition numbers, and homomorphisms between Specht modules.

３．研究の方法

Unfortunately, many research visits and conferences had to be cancelled or postponed due to COVID-19. This greatly slowed down the amount of collaboration, discussion and knowledge exchange that could take place, which in turn slowed down our progress with making progress in type C.
However, some results have been obtained, and we summarise our completed results and ongoing projects in the next section.

４．研究成果

While unable to travel internationally, we had a domestic collaboration with Susumu Ariki (Osaka University) in which we studied an important question for type A Hecke algebras, which also arise as type A cyclotomic KLR algebras in level 1. This project investigated the Schurian-finiteness of blocks of Hecke algebras, in other words we studied which blocks had infinitely many isomorphism classes of Schurian modules. Schurian-finiteness is known to be equivalent to tau-tilting finiteness, an area of intense study in recent years. However, these studies tend to focus on basic algebras, and the basic algebras of Hecke algebras are not known in most cases. Thus the existing methods are largely unable to study Hecke algebras, and we developed new tools for determining that blocks of Hecke algebras are Schurian-infinite, using their graded decomposition numbers. We finished a preprint in which we proved that all blocks of weight 2 and 3 are Schurian-infinite in any characteristic, as are all principal blocks of higher weight. In a follow-up paper, we extended this with Sinéad Lyle to determine that all blocks of weight at least 2 are Schurian-infinite in any characteristic, and these papers have since been merged into one paper containing the more

complete result. This paper – arXiv:2112.11148 – has been submitted to a top-tier general interest journal, and we are awaiting the journal's decision.

Interesting questions naturally arise from this paper, such as extensions to higher levels, which include the Iwahori–Hecke algebras of type B, which we are now beginning to investigate, and extensions to the type C cyclotomic KLR algebras, which are much harder since our graded decomposition numbers method is unlikely to work there. Even in cases where it might, the graded decomposition numbers remain largely unknown in type C.

Our main type C study during this Kakenhi project has been that of determining graded decomposition numbers. In joint work with Chris Chung and Andrew Mathas, we have developed two slightly different algorithms for calculating graded decomposition numbers for the type C cyclotomic KLR algebras. In level 1, we have calculated all graded decomposition for matrices up to rank 12, and even give the characteristic 0 submodule structure of Specht modules up to rank 10. Along the way, we find several interesting results. Firstly, our matrices verify that these algebras are indeed indecomposable up to rank 12, confirming the expected block classification for these algebras. Secondly, we find the smallest example of a characteristic 0 graded decomposition number that does not match the corresponding canonical basis coefficient in this type. In other words, we find the first projective indecomposable module that is not the image of a canonical basis vector under categorification. Thirdly, we find an example of a graded decomposition number in negative degree occurring in characteristic 0, which cannot happen in type A, where the grading filtration and radical filtration are closely connected. Fourthly, we find that the graded decomposition matrices appear to be characteristic-free if the defect of the block is less than 4, and in defects 4 and 5 we have only seen the composition multiplicities increasing in characteristic 2. Many questions remain, and we hope that our data will help us, and other researchers, make progress in studying the type C KLR algebras. We expect to complete and submit our paper by the autumn of 2023.

Besides the above projects, my PhD student Berta Hudak and my postdoc Christopher Chung have also been studying these type C algebras under my supervision, and have determined the representation type for blocks of level 1 cyclotomic quotients. This project resulted in a paper that has been submitted to a journal, and can be found as arXiv:2304.10184.

I am also supervising my PhD student Martín Forsberg Conde in a project which attempts to construct homomorphisms between Specht modules over type C cyclotomic KLR algebras, giving an analogue of a famous result of Carter and Payne. This result has a type A KLR algebra analogue by Lyle and Mathas, and Conde is largely following their strategy, although the type C situation appears to be significantly more difficult.

５．主な発表論文等

〔雑誌論文〕　計2件（うち査読付論文　2件／うち国際共著　2件／うちオープンアクセス　2件）

１．著者名 Robert Muth, Liron Speyer, Louise Sutton	４．巻 26
２．論文標題 Decomposable Specht modules indexed by bihooks II	５．発行年 2021年
３．雑誌名 Algebras and Representation Theory	６．最初と最後の頁 241～280
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s10468-021-10093-3	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Can Mahir Bilen、She Yiyang、Speyer Liron	４．巻 32
２．論文標題 Strong Gelfand subgroups of $F \wr S_n$	５．発行年 2021年
３．雑誌名 International Journal of Mathematics	６．最初と最後の頁 2150010～2150010
掲載論文のDOI（デジタルオブジェクト識別子） 10.1142/S0129167X21500105	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

〔学会発表〕　計9件（うち招待講演　8件／うち国際学会　0件）

１．発表者名 Liron Speyer
２．発表標題 Semisimple Specht modules indexed by bihooks
３．学会等名 London Algebra Colloquium（招待講演）
４．発表年 2021年

１．発表者名 Liron Speyer
２．発表標題 Schurian-infinite blocks of type A Hecke algebras
３．学会等名 Physical Algebra and Combinatorics Seminar, OCAMI（招待講演）
４．発表年 2022年

１．発表者名
Liron Speyer

２．発表標題
Semisimple Specht modules indexed by bihooks

３．学会等名
Algebraic Lie Theory Seminar, University of Colorado Boulder（招待講演）

４．発表年
2021年

１．発表者名
Liron Speyer

２．発表標題
Semisimple Specht modules indexed by bihooks

３．学会等名
London algebra colloquium（招待講演）

４．発表年
2021年

１．発表者名
Liron Speyer

２．発表標題
Schurian-infinite blocks of type A Hecke algebras

３．学会等名
York Algebra Seminar, University of York（招待講演）

４．発表年
2022年

１．発表者名
Liron Speyer

２．発表標題
Schurian-infinite blocks of type A Hecke algebras

３．学会等名
MSJ Autumn Meeting 2022, Hokkaido University

４．発表年
2022年

１．発表者名
Liron Speyer

２．発表標題
Schurian-infinite blocks of type A Hecke algebras

３．学会等名
CART 2022, University of Tsukuba（招待講演）

４．発表年
2022年

１．発表者名
Liron Speyer

２．発表標題
The representation theory of type A Iwahori-Hecke algebras II

３．学会等名
Silver workshop V: Complex Geometry and related topics（招待講演）

４．発表年
2022年

１．発表者名
Liron Speyer

２．発表標題
Graded decomposition matrices for type C KLR algebras

３．学会等名
Representation Theory, Combinatorics and Geometry, National University of Singapore（招待講演）

４．発表年
2022年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

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６．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考
研究協力者	有木　進 (Ariki Susumu)	Ｏｓａｋａ　Ｕｎｉｖｅｒｓｉｔｙ・Emeritus Professor (14401)	

6．研究組織（つづき）

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考
研究協力者	Ｌｙｌｅ　Ｓｉｎｅａｄ (Lyle Sinead)	Ｕｎｉｖｅｒｓｉｔｙ　ｏｆ　Ｅａｓｔ　Ａｎｇｌｉａ・ Associate Professor	
研究協力者	Ｍａｔｈａｓ　Ａｎｄｒｅｗ (Mathas Andrew)	Ｕｎｉｖｅｒｓｉｔｙ　ｏｆ　Ｓｙｄｎｅｙ・Professor	
研究協力者	Ｃｈｕｎｇ　Ｃｈｒｉｓｔｏｐｈｅｒ (Chung Christopher)	ＯＩＳＴ・Postdoctoral Scholar (38005)	

7．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計0件

8．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関