【Grant-in-Aid for Scientific Research (S)】

Broad Section B

Title of Project : Fusion of Birational Geometry and Theory of Periods; Deepening and Exploration of Mirror Symmetry Mathematics

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Research Project Number : 21H04994 Researcher Number : 50314290
Term of Project : FY2021-2025 Budget Allocation : 86,700 Thousand Yen
Keyword : Complex Geometry, Birational Geometry, Mirror Symmetry

【Purpose and Background of the Research】

A geometric object has quantities reflecting its characteristics. For a rectangle, (height) × (width) and (height) ÷ (width) represent its size and shape. For another rectangle whose width is the inverse of the original one, its size and shape become the original shape and size. One has a pair of rectangles replacing the role of the size and shape. Mirror symmetry is, such as in this example, a symmetry exchanging two kinds of geometric features.

More precisely, it is known as an equivalence between two topological string theories called A- and B-models, associated to symplectic geometry and algebraic geometry, which yields many important qualitative and quantitative conjectures, relates wide range of mathematics and deepen classical one along with new findings. It is quite important and interesting to elucidate the mathematical truth behind, with ``physical ideas, objects and methods".

There are two particularly important issues; proof of Homological Mirror Symmetries, an equivalence between categories associated to A- and B-models, and a derivation of the classical one from HMS, an equivalence between Gromov-Witten theory and the deformation theory. The latter is to develop the theory of B-periods for categories via primitive forms and the theory of A-periods via global structures on the space of Bridgeland's stabilities. Towards these issues, many important results have been obtained.

On the other hand, for a further understanding of birational geometry, especially, the minimal model theory, studies by categorical and Hodge theoretical methods have been extensively done. Recently, there appeared a new approach to study derived categories, the categorical dynamics, which also shows the importance of such methods.

By fusion of birational geometry and theory of periods, we will deepen understanding of mirror symmetry phenomena, contribute to traditional important problems and explore new mathematics. We will drastically develop previous results and methods on the following "Academic questions":

1. How much geometric information do triangulated (or derived) categories have?
2. Is it possible to classify algebraic varieties (or derived categories)?
3. What is the geometric/integrable structure that governs flat (Frobenius) structures?

【Research Methods】

Settlement of various problems in the following:

1. Studies of the space of Bridgeland stability conditions and the categorical dynamics based on ideas from theory of periods.
2. Studies of birational geometry, especially the minimal model theory, by categorical and Hodge theoretical methods.
3. Further understanding of period mappings relating Gromov-Witten theory, deformation theory and Weyl group invariant theory.

The plan will be carried out in the following way:

① Individual and joint research by our research system, consisting of research team members, and cooperation researchers (collaborators).
② The activation and the further development by the employment of post-doctoral fellows.
③ Research exchanges by regularly organized seminars, workshops and annual conferences.

【Expected Research Achievements and Scientific Significance】

Not only large contribution in the state-of-the-art studies of mirror symmetry, but our research will also lead to a lot of new knowledge in traditional problems with a history of more than 100 years.

Provision of invariants and methods to birational geometry and elucidation of the relationship among discrete groups, singularities, finite dimensional algebras, root systems, etc., are expected.

Moreover, through our research, we inherit mathematical tradition in the research areas to the next generation and disseminate excellent research from Japan.

【Publications Relevant to the Project】

· KIKUTA Kohei, OUCHI Genki, TAKAHASHI Atsushi, Serre dimension and stability conditions, Mathematische Zeitschrift (2021), published online (March 4, 2021). doi:10.1007/s00209-021-02718-6
· Introduction to Primitive Forms and Mirror Symmetry, Iwanami Studies in Advanced Mathematics, Iwanami Shoten, Publishers, April 9, 2021.

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