【Grant-in-Aid for Scientific Research (S)】

Mathematical innovations woven by singularity theory and geometric topology

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Purpose and Background of the Research

●Outline of the Research

Our research on global theory of singularities (a theory that attempts to capture the overall essence of a figure or space by focusing on the places where its observed data behaves in a particular way, so-called singularities) based on concrete constructions is beginning to attract worldwide attention. In this study, we combine the ideas of constructive geometric topology with singularity theory to dramatically develop the singularity theory of differentiable maps, and then feed back the methods of singularity theory to geometric topology to solve major problems concerning manifolds, which are extremely important in mathematics.

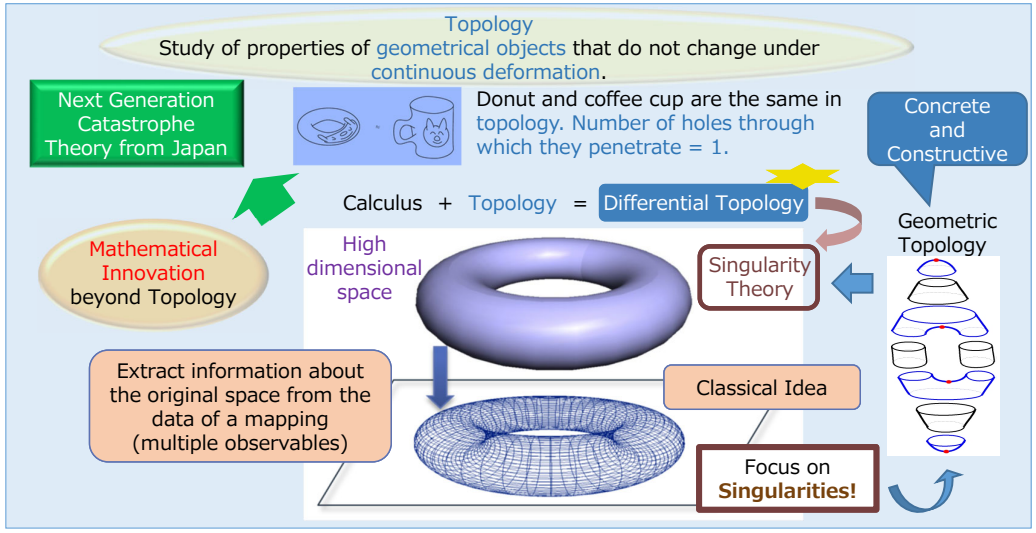

Figure 1. Overall Picture of the Research

●Purpose of the Research

In recent years, a lot of important and novel results have been obtained in low-dimensional geometric topology by essential use of geometric structures. By bringing such powerful geometric ideas and rich theories of low-dimensional topology to the singularity theory, our research aims to innovate existing concepts, formulations, and methods, and to make a quantum leap forward in global singularity theory. Furthermore, we open a new way to geometric topology from singularity theory, and solve important problems. Through such innovative research, we aim to create a new research field beyond topology, "next generation catastrophe theory," and to bring about new developments in topology as an academic field through innovative applications to different fields, such as artificial intelligence, etc. In this way, we make a significant contribution to society through mathematical innovations.

●Methods of the Research

We make full use of singular fiber theory (a theory that studies the shape of places with the same data values), created and established by Principal Investigator Saeki, singular geometric structures (structures that allow the disruption of a clean sequence of data), mapping class groups and monodromy (useful methods for studying data torsion). Using the concept of "charts," which visualize braid groups and mapping class groups, we study in a unified manner the theory of algebraic invariants, which until now have not been well understood geometrically.

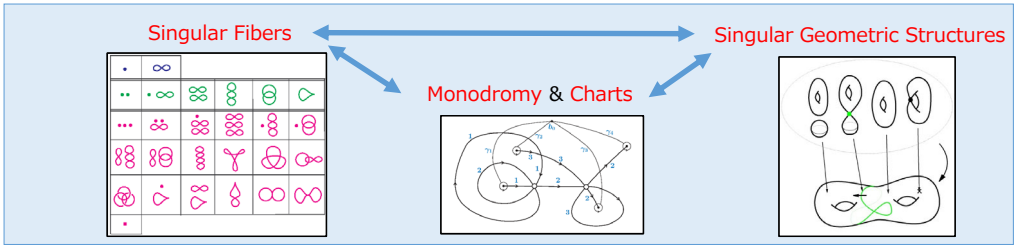

Figure 2. Diagram of Research Methods

Expected Research Achievements

●Originality of the Research

- We establish a theory that treats singular points/fibers of differentiable maps in a unified manner in terms of geometric topology, which has not been developed before, and in this sense our research is groundbreaking.
- We concretely construct polynomials that realize certain manifolds, and then, using singularity theory, we reveal the path to the solution of the still unresolved 4D smooth Poincaré conjecture with innovative ideas such as using algebraic invariants of the polynomials reflecting the structures of the manifolds.
- In addition to constructing Vassiliev-type invariants of manifolds based on new ideas, this research also constructs invariants of 4D manifolds based on singular geometric structures and clarifies various structures of manifolds, which are original in nature.

Furthermore, we attempt to apply such research to important problems in various scientific fields. Generic maps exist abundantly among all manifolds of any dimension. Since data can be approximated by such maps, singularity theory of maps is versatile, and there is a good possibility of obtaining results for general dimensions, not limited to special dimensions. If a new research area is created through this research, it will make a significant contribution to various scientific fields, and eventually to the society.

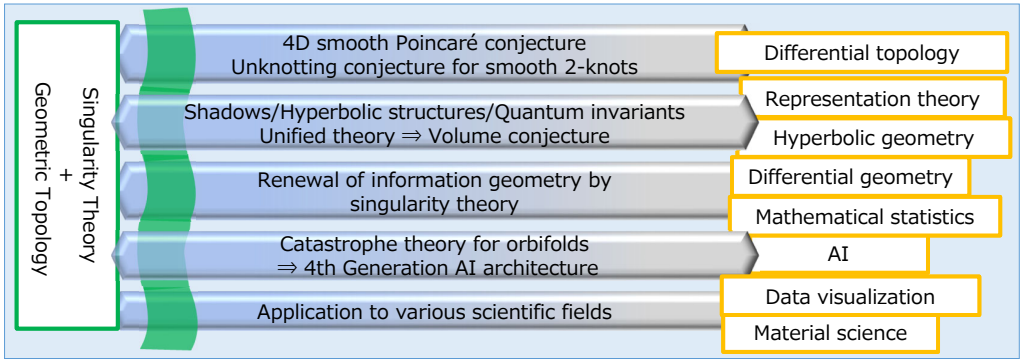

Figure 3. Mathematical innovations woven by singularity theory and geometric topology

Homepage Address, etc.	https://imi.kyushu-u.ac.jp/~saeki/index.html