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研究課題名（和文）Combining different types of data for geophysical inverse problems: Theory and applications

研究課題名（英文）Combining different types of data for geophysical inverse problems: Theory and applications

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研究成果の概要（和文）：この研究プロジェクトは、異なるタイプのデータを組み合わせることにより、EIV、乗法的な誤差と離散モデルにおける未知パラメータを推定する逆問題が研究する。研究成果として国際学術誌で9論文を発表した。いくつかの国際シンポジウムで成果を発表した。具体的には、乗法誤差モデルの偏り補正する最小二乗法を開発した。ディスクリート離散モデル格子基底縮小の品質評価を行った。格子基底縮小の品質を直接にグラム・シュミット係数の確率分布によって影響されることがわかった。EIVモデルについてはN－補正の推定量を構築した。EIVモデルにおける分散成分は、特定の条件下で推定ができないことを証明し、不安定であることもある。

研究成果の概要（英文）：This project focuses on inverse problems to estimate the unknown parameters in errors-in-variables (EIV), multiplicative noise and discrete tomography models by combining different types of data. We achieve a number of new results and publish 9 papers in international journals. The applicant is invited to report the achieved results at several international conferences. We develop new LS-based estimators to reconstruct multiplicative error models and conduct quality evaluation of lattice basis reduction for discrete tomography, which can be directly affected by the probability distribution of the reduced Gram-Schmidt coefficients. We propose bias-corrected LS and N-calibrated estimators to estimate the EIV parameters, which are as good as total LS, but require much less computation. We prove that the variance components in an EIV model can be inestimable and unstable. We investigate how EIV can affect the parameter estimation and the weighting factors for different types of data.

研究分野： 測地学

キーワード： parameter estimation　inverse problems　errors-in-variables　multiplicative noise　integer estimation　variance components

１．研究開始当初の背景

(1) Inverse ill-posed problems has been based on the implicit assumption that the weighting factors of measurements are known. At the present, more and more measurements are routinely collected with different types of sensors or instruments to solve inverse ill-posed problems in Earth Science (and beyond) for the most accurate and high resolution Earth models. Thus, combining different types of data with heteroscedastic nature and/or different accuracies has been becoming more and more important and essential to accurately estimate the unknown parameters of inverse problems.

(2) The weighting factors of data of different types have been either assumed to be known or empirically selected in all joint inversions (of geophysical inverse ill-posed problems). Although combining different types of data to accurately estimate the parameters in an inverse problem is of significantly increasing importance in science and engineering, extremely little attention has been done to develop a solid foundation for use in any inverse problems. Although covariance structures with a few unknown parameters have been proposed in the statistics literature, these methods simply failed, if the stochastic models of different types of data contain a number of unknown variance components, or equivalently, unknown weighting factors, as has been clearly demonstrated by a great number of simulations.

(3) The major purpose of this research project is to lay a solid mathematical foundation for inverse problems with different types of data, which is urgently demanded and is expected to have widest possible applications in many areas of science and engineering. On the other hand, satellite gravity has found widest possible applications in Earth Sciences, we will further investigate its mathematics for use in highly interdisciplinary subjects.

２．研究の目的

Motivated by the success of this applicant in dealing with different types of data for inverse problems, we aim at developing a systematic solid mathematics foundation for simultaneous estimation of parameters and weighting factors for inverse problems.

(1) Generalized cross-validation and L-curve approach have been widely used to estimate the unknown parameters in inverse problems. In general, one assumes that the elements of the design matrix are free of random errors. In practice, such elements are often obtained either directly from measurements and/or indirectly computed from measurements. As important inverse problem models of the research, we focus on an errors-in-variables (EIV) model and discrete tomography, with applications to GNSS. In such cases, the estimation of parameters will be biased, depending on the size of EIV random errors. To correctly reconstruct errors-in- variables models, we have to correctly determine the weighting factors of different types of data, which may be collected, for example, with different types of sensors with different variance components;

(2) Although the regularization parameter is of crucial importance for the quality of solution to an inverse problem of EIV type, it can be erroneously determined due to incorrect weighting factors of different types of data. On the other hand, for inverse problems of EIV type, we will investigate how the calibration of the normal matrix could be used for regularization. We will develop a new N-calibrated solution-quality-based method to unbiasedly reconstruct EIV models;

(3) As an important element of this research project, numerical confirmation will be crucial to show the power and significant performance of the developed theory and methods. Thus we will scientifically design a variety of experiments to fairly compare the developed methods and theory through numerical simulations; and finally,

(4) Since satellite gravity is one of the geodetic hot topics, with profound applications to geophysics, water resource survey, and Earth environmental monitoring, we will further develop theory and methods for accurate and super-resolution gravity models.

３．研究の方法

To systematically develop the theory and methods for solving joint inverse problems with different types of data, as first originated from Xu et al. (2006) and Xu (2009), we plan to investigate several major problems on the topic. We will simultaneously focus on theoretical development and demonstrate the developed theory and methods with EIV models.

Our basic approach to addressing and solving inverse problems with data of different types and different accuracy would consist of the following aspects: least squares estimation, bias analysis, determination of unknown weighting factors, and proposing an N-calibrated estimator for regularization.

Finally, we will then work on high-precision and high-resolution satellite gravity models.

４． 研究成果

(1) 離散 tomography を含める逆問題は重要な課題として研究を行った。離散 tomography について、我々は整数最小二乗解をより速く得るために格子基底縮小の新しいアルゴリズムを開発し、更に６つの比較基準に基づいて、今までよく使用されていたアルゴリズムと比較実験研究を行った。格子基底縮小についての研究実績は以下の通りである：我々は開発した正定値二次形式の改良された LLL アルゴリズムを格子ベクトル基底へ適用した。格子基底縮小の直交性を評価するために、我々は更に縮小基底の最小角度を評価対象として提案した。これまでの知見からも、縮小基底の最小角度は格子基底縮小の直交性基準よりも魅力的な評価対象である。なお、LLL アルゴリズムとその変形はこれまで広く使用されているが、これらの格子基底縮小アルゴリズムの有効性の実験的シミュレーションが行われたのは最近であり、しかもそれらは Hermite 係数（図１）、実用的なランニング形態と縮小グラムーシュミット係数に限定されている。我々は、新しいアルゴリズムを含めて、LLL アルゴリズムと deep-insertion のあるアルゴリズムなどについて、6 つの格子基底縮小品質基準を用いて大規模な実験調査研究を行った；シミュレーションにより、新しいアルゴリズムは他の既存の４つの LLL 関連したアルゴリズムより、比較用の 6 つの格子基底縮小品質基準にて、全てより優れた結果を与えることが判明した；また、シミュレーションから得た 6 百万個以上の縮小グラム - シュミット係数を利用し、格子基底縮小アルゴリズムの統計的特徴について研究を進めた。実験結果により、縮小グラムーシュミット係数は均一分布ではないことも明らかになった。それらのゼロに近い小さな係数は多ければ多い程、また、0.5 と-0.5 の両端部付近の大きい係数は少なければ少ない程、格子基底縮小アルゴリズムの縮小効果がよいことも初めてわかった（図２）。これらの研究成果は Eurasip J Adv Signal Proc で公表され、また、アメリカ数学会２０１４年大会で招待研究発表を行った。

(2) 宇宙測地観測技術は全て電磁波を利用し、観測をしてきたが、これらの観測データは今迄加法的誤差として扱いました。ただ、GPS/VLBI/SLR などの Baselines、InSAR と LiDAR などの観測精度式により、これらの観測データ誤差の中に乗法的な成分がはっきりあることがわかった。乗法的な観測誤差を含める逆問題について正しく評価する為に、我々は偏り補正最小二乗法を開発した。また、乗法的な観測誤差の variance of unit weight の推定量を導いて統計的な特徴を解析した。数値シミュレーションを利用して、これらの推定量の評価も行った。観測データの非線形性は最小二乗推定量の偏りをいつも生み出すのに、variance of unit weight 推定量への影響はほとんどないことがわかった。研究成果はヨーロッパの地球科学連合２０１５年大会で招待研究発表をし、国際学術誌 Sensors と Acta Geod Geophys で公表された。

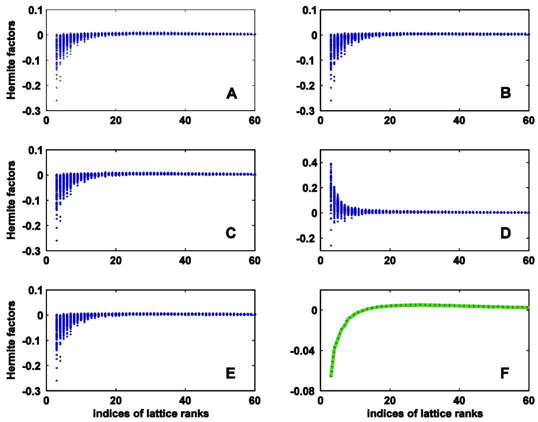

図１：The Hermite factors of the 10,000 random examples (in logarithm) after the reductions by LLL (panel A), DEEP (panel B), SLLL (panel C), VLLL (panel D) and PLLL (panel E). Shown in panel F are the Hermite factors of the random problems (in logarithm with symbol +) and the mean values of η for each rank of lattice, with the green line for PLLL and the dashed red line for DEEP.

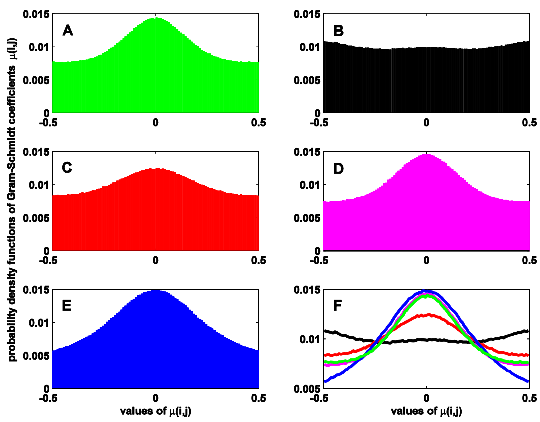

図２：The pdf histograms of the Gram-Schmidt coefficients after the reductions. A --- LLL; B --- DEEP; C --- SLLL; D --- VLLL; and E --- PLLL. In order to show a direct comparison of these methods, we have also plotted all the pdf curves together in panel F: the green line --- LLL; the black line --- DEEP; the red line --- SLLL; the pink line --- VLLL; and the blue line --- PLLL.

(3) For inverse EIV models, we have proved that the variance components in the EIV stochastic models are not estimable, if all the elements of the design matrix A are random without any functional constraints and can be classified into, at least, two groups of data of the same accuracy. The same is true for the EIV model of regression type. This result about inestimability implies statistically that we cannot do anything to gain any knowledge on such EIV stochastic models. From this point of view, to gain knowledge on the EIV stochastic models, one will have to directly collect repeated measurements and/or alike of the elements of A for such a stochastic model. Otherwise, if the variance components are estimable, we derive the MINQUE estimates of the variance components in EIV models. For the measurements with different accuracy in different groups, we have derived the componentwise positive estimates of the variance components. The variance component estimation in the EIV model may be unstable, however, as confirmed by the simulated numerical examples. Finally, we have worked out the finite sample biases of the variance components, if they are estimable. As a result of equation instability, the biases of the estimated variance components could be significantly amplified due to a large condition number.

(4) While the EIV model has been widely applied in practice, total least squares (TLS) and weighted LS methods are known to produce almost negligible differences for some problems. This is particularly true in geodetic coordinate transformation. We have investigated the effects of the random errors of the design matrix on the weighted LS estimates of quantities of interest, in particular, the model parameters, the variance-covariance matrix of the estimated parameters and the variance of unit weight. By taking the bias and the variance-covariance matrix into account, we can further compute the mean squared error matrix for the weighted LS estimate. The theoretical analysis of bias has shown that the effect of EIV on adjustment depends on the random design matrix itself, the variance-covariance matrix of the EIV errors, the measurement weighting matrix and the model parameters. Roughly speaking, the bias of the weighted LS estimate would depend on two factors: the model parameters and the values of the noise-to-signal ratios. If A contains no random errors, then the bias of the weighted LS estimate becomes zero. The interdisciplinary example of climate change has revealed a significant impact of the EIV on the weighted LS estimate. The biases of both parameters are even at the same order of magnitude as the parameters themselves.

(5) We propose calibrating the normal matrix by removing the bias of the random normal matrix and accordingly constructed an N-calibrated weighted LS estimate. The simulations have clearly shown that both estimators work well to reduce the bias of the weighted LS estimate. If the signal-to-noise level is sufficiently large, the bias of the weighted LS estimate can be removed almost completely, without loss in accuracy. The simulations show surprisingly that the N-calibrated weighted LS estimate is easier computationally and performs even better than the bias-corrected weighted LS estimate in reducing the bias of the weighted LS estimate, though it is intuitively constructed to calibrate the normal matrix only（図３)。

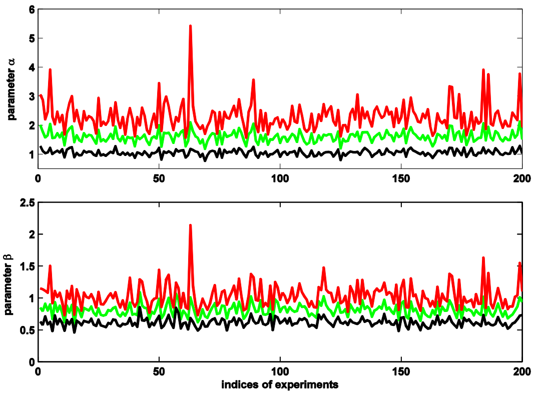

図３：The first 200 examples of the weighted LS estimator (black line), the bias-corrected weighted LS estimator (green line) and the N-calibrated weighted LS estimator (red line) on the basis of the original data from Mann and Emanuel (2006). The true values of two parameters are equal to 2.2 and 1.0, respectively.

(6) The numerical integration method has been used by almost major institutions

worldwide to produce global standard gravitational models from satellite tracking measurements of CHAMP and/or GRACE types. Such Earth's gravitational products have found widest possible multidisciplinary applications in Earth Sciences. The method is essentially implemented by solving the differential equations of the partial derivatives of the orbit of a satellite with respect to the unknown harmonic coefficients under the conditions of zero initial values. From the mathematical and statistical point of view, satellite gravimetry from satellite tracking is essentially the problem of estimating unknown parameters in the Newton's nonlinear differential equations from satellite tracking measurements. We prove that zero initial values for the partial derivatives are incorrect mathematic-cally and not permitted physically. The numerical integration method, as is currently implemented and used, is groundless, both mathematically and physically. This result is an invited talk at the EGU 2015 General Assembly in Vienna.

５．主な発表論文等
（研究代表者、研究分担者及び連携研究者には下線）

〔雑誌論文〕（計　８　件）

①Xu PL, The effect of errors-in- variables on variance component estimation, Journal of Geodesy, 査読有，2016, DOI 10.1007/s00190-016-0902-0

② Shi Y, Xu PL, Comparing the estimates of the variance of unit weight in multiplicative error models, Acta Geodaetica Geophysica, 査読有，2015, 50, 353-363, DOI 10.1007/s40328-014-0096-y_

③ Shi Y, Xu PL, Liu JN, Shi C, Alternative formulae for parameter estimation in partial errors-in- variables models, Journal of Geodesy, 査読有，2015, 89, 13-16, DOI: 10.1007/s00190-014-0756-2

④ Xu PL, Liu JN, Zeng W, Shen YZ, Effects of errors-in-variables on weighted least squares estimation, Journal of Geodesy, 査読有，2014, 88, 705-716, DOI: 10.1007/s00190-014-0716-x

⑤ Xu PL, Liu JN, Variance components in errors-in-variables models: Esti- mability, stability and bias analysis, Journal of Geodesy, 査読有，2014, 88, 719-734, DOI: 10.1007/s00190-014-0717-9

⑥ Shi Y, Xu PL, Peng J, Shi C, Liu JN, Adjustment of measurements with mul-tiplicative errors: error analysis, estimates of the variance of unit weight and effect on volume estimation from lidar-type digital elevation models, Sensors, 査読有，2014, 14, 1249-1266, doi:10.3390/s140101249

⑦ Xu PL, Shi Y, Peng JH, Liu JN, Shi C, Adjustment of geodetic measurements with mixed multiplicative and addi- tive random errors, Journal of Geodesy, 査読有，2013, 87, 629-643, DOI: 10.1007/s00190-013-0635-2

⑧ Xu PL, Experimental quality evaluation of lattice basis reduction methods for decorrelating low- dimensional integer least squares problems, EURASIP Journal on Advances in Signal Processing, 査読有，2013, 2013:137, 10.1186/1687-6180-2013-137

〔学会発表〕（計　８　件）

① Xu PL, A review of integer estimation for GPS, Asian-Pacific GNSS Meeting, Kyoto, Nov 16-19, 2015

② Xu PL, Mathematical foundation for the next generation of global gravity models from satellite gravity missions of CHAMP/GRACE types, IUGG General Assembly, Prague, Jun 21-Jul 1, 2015

③ Xu PL, Quality evaluation of lattice basis reduction, Chinese Satellite Navi Conference, Xi'an, May 13-15, 2015 (invited)

④ Xu PL, Zero initial partial derivatives of satellite orbits with respect to force parameters nullify the mathematical basis of the numerical integration method for the determination of standard gravity models from space geodetic measurements, European Geosciences Union, Vienna, Apr 12-17, 2015 (invited)

⑤ Xu PL, Least-squares-based adjust-ment of geodetic measurements with multiplicative random errors, European Geosciences Union, Vienna, Apr 12-17, 2015 (invited)

⑥ Xu PL, Experimental quality evalua-tion of lattice basis reduction methods for discrete tomography, American Mathematical Society Joint Meeting, Baltimore, Jan 15-19, 2014 (invited)

⑦ Xu PL, Variance components in errors-in-variables models: estimability, stability and bias analysis, VIII Hotine-Marussi Intl Sym, Rome, June 17-21, 2013 (invited)

⑧ Xu PL, Fast integer estimation methods for GPS ambiguity resolution:

review and improvement, European Geoscience Union General Assembly, Vienna, April 7-12, 2013 (invited)

〔図書〕（計　１　件）

① Xu PL, Mixed integer linear models, in: Handbook of Geomathematics, W. Freeden, Z. Nashed, T. Sonar (2nd ed), 2015, pp.2405-2451, Springer, Berlin.

〔産業財産権〕

○出願状況（計　0　件）

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権利者：
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番号：
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〔その他〕
ホームページ等

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