【Grant-in-Aid for Scientific Research (S)】

Developments of theory of hyperplane arrangements and application to black box of AI

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Project Information	Project Number : 25H00399　　Project Period (FY) : 2025-2029 Keywords : Hyperplane arrangements, Logarithmic derivations, Symmetry, Combinatorics, AI and arrangements

Purpose and Background of the Research

●Outline of the Research

Points, lines and so on divide ambient spaces, and their generalizations to higher dimensional spaces are so called hyperplane arrangements. Their origins are root systems, and they are studied from algebra, geometry, combinatorics, representation theory and so on (Fig. 1). On the other hand, arrangements ``divide'' spaces, which is a principal in the image recognition AI. In this research, from the first viewpoint, we unite several theory on arrangements in which logarithmic derivations (algebraic aspects) are centered. For the latter, by studying arrangements which are produced by AI, we understand ``understanding of AI'' mathematically.

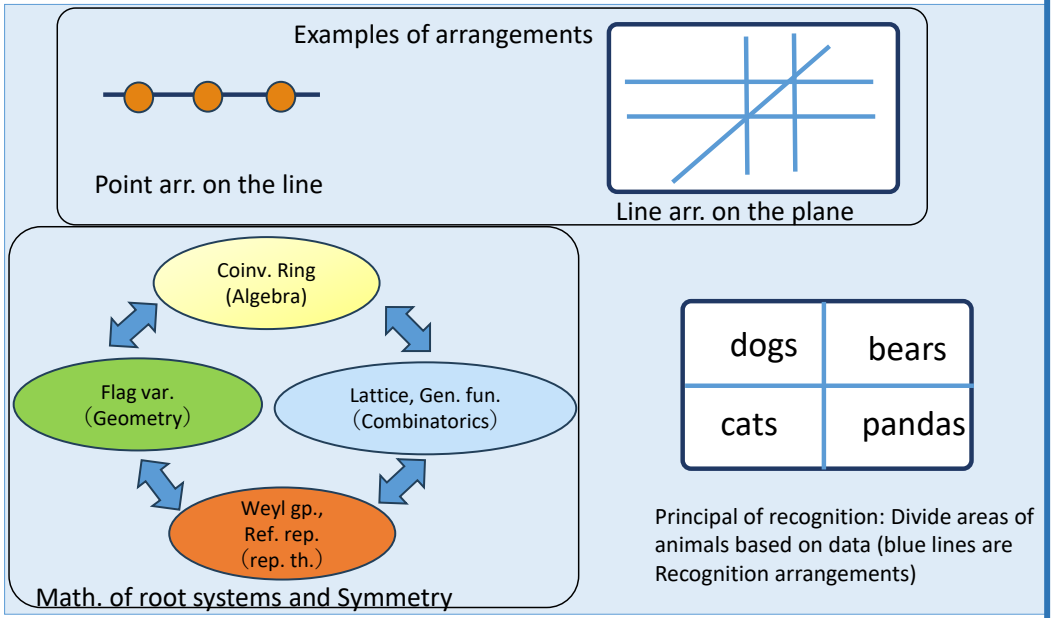

Figure 1. Examples of arrangements and basics of the research

●AIM of the Research

Arrangements are finite sets of hyperplanes in a vector space, which are simple geometric objects. Easy examples are points on the line or lines in the plane (Fig. 1). Recently, their algebraic aspects are drastically developed, and we deepened the Solmon-Terao theory. As the root systems, which are the origin of arrangements, were related to algebra, geometry and combinatorics through the (co)invariant theory, we expect to do the same for arrangements through logarithmic derivation modules and Solomon-Terao theory to make a new research area. At the same point, we study recognition AI through arrangements which are made by AI to do the recognition.

Now, how AI is doing image recognition is in the mist, and that is a big problem. So in this research, by studying the recognition arrangements made by AI mathematically, we compare AI and understand the way of understanding by AI quantitatively (Fig. 2). Also, we study overlearning of SVM and adversarial examples to AI in cryptography and coding theory by using arrangements.

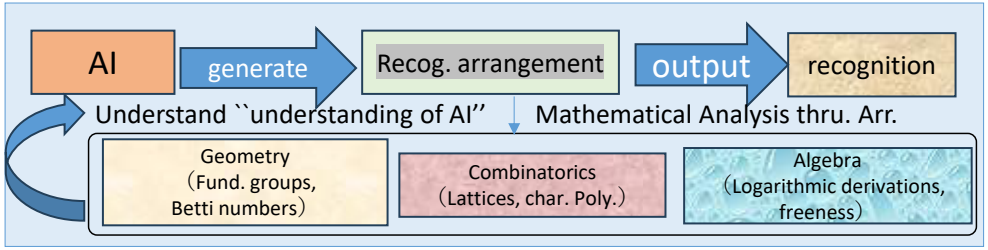

Figure 2. Image Diagram of application research

Expected Research Achievements

●Explicit AIM of the Research

1. Complete the Solomon-Terao bi-polynomial theory by using logarithmic derivations.
2. Understand theory 1 in terms of geometry, rep. theory, and combin.. Explicitly, by geometry of Hessenberg and critical varieties, rep. theory of Liouville complexes, and quasi-invariants, and combinatorics of superspace invariants (Fig. 3, dotted lines).
3. Unite the theories in 2 by using Solomon-Terao theory as a hub (Fig. 3 green part).
4. Analyze mathematical invariants of recog. arrangements made by AI to understand the understanding, explanation, comparison and control of AI mathematically.
5. Apply the results in 4 to cryptography and coding theory.

●Originality of the Research

1. Theoretical aspects. Starting from arrangements and using them as hubs, the trial to understand several mathematics uniformally and to create a new research area. Also, the trial to solve several classic problems by using these new theory.
2. Application aspects. Try to understand the way of understanding by AI by using hyperplane arrangements mathematically, and quantitively.

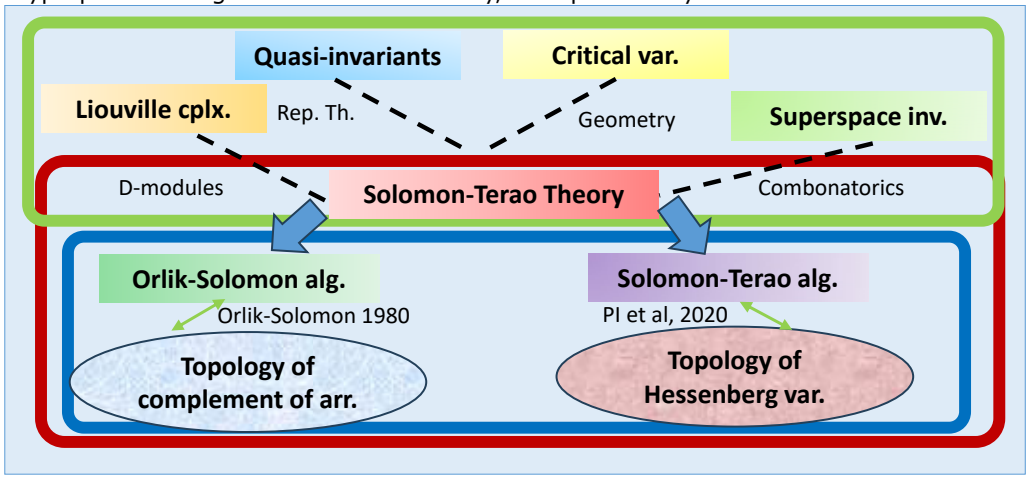

Figure 3. Theoretical Research (Blue:Done Red:Almost done Green:New theory

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