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研究課題名（和文） Algebraic Complexity Theory: New Approaches and Algorithmic Applications

研究課題名（英文） Algebraic Complexity Theory: New Approaches and Algorithmic Applications

研究代表者

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交付決定額（研究期間全体）：（直接経費）　　9,600,000円

研究成果の概要（和文）：　本研究課題では、代数的計算量理論に基づき、理論計算機科学の諸問題を解くための新しい手法を開発した。研究期間内に得た主な成果は次のように大別できる。

まず、代数的問題を解く新しいアプローチを開発し、長方形行列積アルゴリズムの高速化に成功した。次、代数的手法を新たな理論計算機科学の問題に応用することができた。特に分散計算においては、様々な問題に対して、高速な分散アルゴリズムを構築した。最後に、グラフ問題に対して、従来のアルゴリズムより高速な量子アルゴリズムの開発にも成功した。

研究成果の学術的意義や社会的意義

行列積は情報処理の普遍的な計算である。本研究で開発した行列積アルゴリズムは世界最高速であり、すでに理論計算機科学に広く使われており、今後もアルゴリズムの設計の重要なツールになると期待できる。分散計算における新アプローチは、汎用性が高く、分散計算の基幹技術になる可能性が高い。開発した量子アルゴリズムも、重要なグラフ問題を解くものであり、量子計算の優位性の確立に向けて大きな役割を果たすと期待できる。

研究成果の概要（英文）：In this research project we developed new techniques based on algebraic complexity theory to solve problems from theoretical computer science. Our main achievements are as follows.

We first developed new approaches to solve algebraic problems, and constructed faster algorithms for rectangular matrix multiplication. We then showed how to apply in a novel way algebraic techniques to several problems from computer science. In particular, via this approach we designed fast distributed algorithms for various computational problems over networks. Finally, we designed several quantum algorithms for graph problems that are faster than the best known algorithms.

研究分野： 理論計算機科学

キーワード： 計算量理論　アルゴリズム　代数的問題　量子計算

１．研究開始当初の背景

Matrix multiplication is one of the most fundamental tasks in science and engineering. In particular, matrix multiplication is a crucial tool in computer science, as the best known algorithms for central problems in several branches of computer science rely on fast algorithms for matrix multiplication. The computational complexity of matrix multiplication is nevertheless still unsettled, and remains one of the major open problems in theoretical computer science.

The computational complexity of matrix multiplication and related algebraic problems has been studied by both mathematicians and computer scientists within the field of **algebraic complexity theory** since the 1960s. These researches lead to the celebrated algorithm for matrix multiplication by Coppersmith and Winograd (Journal of Symbolic Computation, 1990), which has then been unbeaten for more than two decades until an improvement was finally obtained by Vassilevska-Williams (STOC, 2012).

While matrix multiplication has been used for a long time as a crucial tool for the design of algorithms in many areas of computer science, **investigating new algorithmic tasks where matrix multiplication can be used to speed-up computation is still a very active area of research**. Recent exciting developments are the new use of algebraic algorithms for matrix multiplication in distributed computing by Censor-Hillel et al. (PODC, 2015) or the application of matrix multiplication in learning theory by Valiant (Journal of the ACM, 2015).

Matrix multiplication has also recently become a central concept for new approaches investigating hardness and equivalence for tractable problems. These investigations essentially started from the surprising results by Williams and Vassilevska-Williams (FOCS, 2010) and Patrascu (STOC, 2010) showing that fundamental problems in theoretical computer science for which no to little algorithmic progress has been done in the past decades (namely, triangle finding in graphs and 3SUM) are equivalent to matrix multiplication (under an appropriate definition of equivalence and an appropriate definition of the matrix product). Similar equivalence results for several other fundamental computational problems have been established in the last few years, and this new research direction has become one of the most active areas in theoretical research on algorithms.

These recent, exciting and successful developments on algorithmic applications of matrix multiplication nevertheless typically use algebraic complexity theory only in a limited way. A more ambitious project is **investigating how deeper concepts and techniques from algebraic complexity theory can be exploited** as well. Seminal results in this direction have recently been obtained, in particular by Cygan, Gabow and Sankowski (Journal of the ACM, 2015) who obtained new algorithms for several graph-theoretic problems via advanced tools from algebraic complexity, motivating further investigation of this promising research approach.

２．研究の目的

The purpose of this research project was to further **investigate the role of algebraic approaches for algorithmic problems in theoretical computer science**. The main objective was to develop and combine in a novel way two lines of research that have mostly been studied independently so far: theoretical research on the complexity of matrix multiplication and algebraic problems (a field often called “algebraic complexity theory”) on one side, and recent works by the algorithm community on applications of matrix multiplication on the other side.

The goal of this research proposal was to further investigate both **the role of matrix multiplication and algebraic approaches for algorithmic problems** in computer science, and the **interactions between these research areas**. This project has been implemented along the three directions described below.

A. *Algebraic complexity theory*: *new approaches for algebraic problems*

The main open problem in the field is naturally to construct better algorithms for matrix multiplication. To make progress on this very challenging problem, our proposal was to first precisely **identify viable and promising approaches**. The first concrete goal here was to characterize, qualitatively and quantitatively, the limitations of existing approaches to matrix multiplication. Based on these characterizations, the second stage of this research focused on the most promising approaches for matrix multiplication and other algebraic problems, and exploring their potentials.

B. *Applications of algebraic complexity*: *Development of new algorithmic applications*
This part of the research aimed both at developing **novel applications of matrix multiplication and other algebraic techniques** to existing non-algebraic algorithmic problems (e.g., graph-theoretic problems) and identifying **novel algorithmic tasks for which matrix multiplication can be exploited**. It also investigated how matrix multiplication and other algebraic techniques can be used to **reduce costs other than the time complexity**. Special attention focused on the space complexity of algorithm tasks, the communication complexity of protocols and the round complexity of distributed algorithms for fundamental computational tasks.

C. *Applications of algebraic complexity*: *Theory of hardness and equivalence*
The goal of the third direction of this research was to confirm and put into new light the central role of algebraic problems in the novel approach (hardness and equivalence of tractable problems) discussed in the previous page. Concretely, the goal was to **identify computational problems in which instances of matrix multiplication and other algebraic structures can be embedded** and consequently classify their hardness. In comparison to most prior works, such classifications have been investigated not only with respect to time complexity but also with respect to other complexity costs such as the space complexity and the query complexity, and **in other models of computation such as distributed computing or quantum computing** as well.

３．研究の方法

The main originality of this research program was the **interaction between the three directions described above**. As already mentioned, research on algebraic complexity theory (Direction A) and research on applications of matrix multiplications (related to Directions B and C) have until recently mostly been done independently. One of the main goals of this project was to **combine them in a novel way,** while developing each of these three directions. A schematic representation of this interaction is given in the following figure.

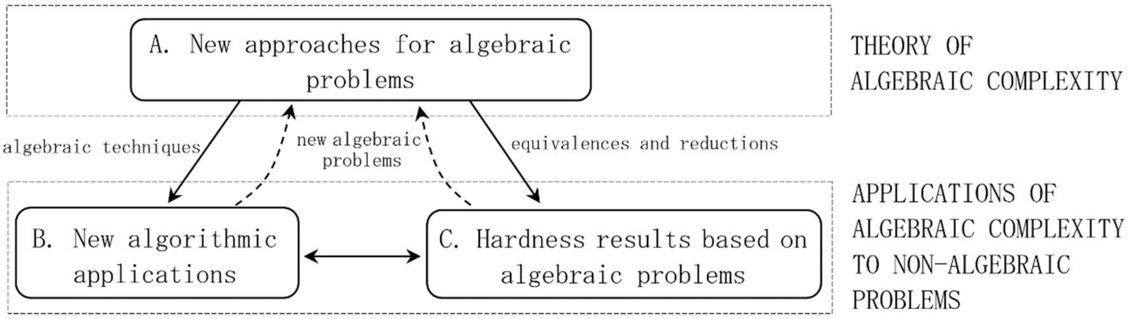


Figure: Overview of our research proposal

Let us now describe how research was conducted during the four years of this project.

First, progress has been made on each of the three directions of the project, i.e., on both the theory of algebraic complexity (in particular, on the complexity of matrix multiplication) and on its applications to central non-algebraic problems in theoretical computer science. Secondly, and more conceptually, this project discovered new relations between these two areas of research that may be crucial for further progress on both areas.

More concretely, this research project was immediately launched in FY 2016 by **developing and extending the techniques introduced in the most recent prior works**. These preliminary investigations directly lead to new results showing the limitation of current techniques, and clarifying where new approaches needed. From FY 2017, we then used these findings to **progressively investigate new approaches and new targets** for each of Directions A, B and C of the research plan, and simultaneously start a systematic **investigation of their interactions.** Research from FY 2018 focused on **combining these directions in a novel way** to both make progress on each area and discover new relations between them.

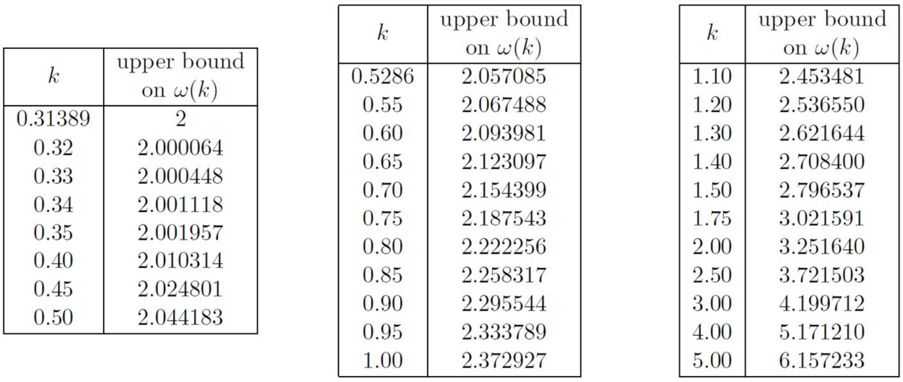
４．研究成果

During the four years of this research project we have obtained results that lead to 17 publications in international conferences and journals. We present below the main results obtained.

Faster algorithms for rectangular matrix multiplication

In the past few years, successive improvements of the asymptotic complexity of square matrix multiplication have been obtained by developing novel methods to analyze the powers of the Coppersmith-Winograd tensor, a basic construction introduced thirty years ago. We have shown how to generalize this approach to make progress on the complexity of rectangular matrix multiplication as well, by developing a framework to analyze powers of tensors in an asymmetric way. By applying this methodology to the fourth power of the Coppersmith-Winograd tensor, **we have succeeded in improving the complexity of rectangular matrix multiplication**.

Let α denote the maximum value such that the product of an $n \times n^{\alpha}$ matrix by an $n^{\alpha} \times n$ matrix can be computed with $O(n^{2+\epsilon})$ arithmetic operations for any $\epsilon > 0$. By analyzing the fourth power of the Coppersmith-Winograd tensor using our methods, we obtained the new lower bound $\alpha > 0.31389$, which improves the previous lower bound $\alpha > 0.30298$ obtained five years ago by Le Gall (FOCS'12) from the analysis of the second power of the Coppersmith-Winograd tensor. More generally, we gave faster algorithms computing the product of an $n \times n^{k}$ matrix by an $n^{k} \times n$ matrix for any value $k \neq 1$. (In the case $k = 1$, we recovered the bounds recently obtained for square matrix multiplication). The following table (Table 3 from [Le Gall and Urrutia, SODA 2018]) gives the numerical values of the exponent of rectangular matrix multiplication (denoted ω(k)) we obtain for several values of k.



k	upper bound on $\omega(k)$
0.31389	2
0.32	2.000064
0.33	2.000448
0.34	2.001118
0.35	2.001957
0.40	2.010314
0.45	2.024801
0.50	2.044183

k	upper bound on $\omega(k)$
0.5286	2.057085
0.55	2.067488
0.60	2.093981
0.65	2.123097
0.70	2.154399
0.75	2.187543
0.80	2.222256
0.85	2.258317
0.90	2.295544
0.95	2.333789
1.00	2.372927

k	upper bound on $\omega(k)$
1.10	2.453481
1.20	2.536550
1.30	2.621644
1.40	2.708400
1.50	2.796537
1.75	3.021591
2.00	3.251640
2.50	3.721503
3.00	4.199712
4.00	5.171210
5.00	6.157233

Table : Our upper bounds on the exponent of the multiplication of an $n \times n^{k}$ matrix by an $n^{k} \times n$ matrix, obtained by analyzing the fourth power of the Coppersmith-Winograd tensor.

These improvements immediately lead to **improvements in the complexity of a multitude of fundamental problems** for which the bottleneck is rectangular matrix multiplication, such as computing the all-pair shortest paths in directed graphs with bounded weights.

These results have been published in the Proceedings of SODA 2018.

Probabilistic logarithmic-space algorithms for Laplacian solvers

A recent series of breakthroughs initiated by Spielman and Teng culminated in the construction of nearly linear time Laplacian solvers, approximating the solution of a linear system Lx=b, where L is the normalized Laplacian of an undirected graph. We have studied the space complexity of the problem. Surprisingly we have been able to **show a probabilistic, logspace algorithm solving the problem**. We further extended the algorithm to other families of graphs like Eulerian graphs (and directed regular graphs) and graphs that mix in polynomial time.

Our approach was to pseudo-invert the Laplacian, by first "peeling-off" the problematic kernel of the operator, and then to approximate the inverse of the remaining part by using a Taylor series. We approximated the Taylor series using a previous work and the special structure of the problem. For directed graphs we exploited in the analysis the Jordan normal form and results from matrix functions.

These results have been published in the Proceedings of RANDOM 2017.

Design of fast classical distributed algorithms for triangle detection

Triangle-free graphs play a central role in graph theory, and triangle detection (or triangle finding) as well as triangle enumeration (triangle listing) play central roles in the field of graph algorithms. In distributed computing, algorithms with sublinear round complexity for triangle finding and listing have recently been developed in the powerful CONGEST clique model, where communication is allowed between any two nodes of the network. We have obtained **the first algorithms with sublinear complexity**

for triangle finding and triangle listing in the standard CONGEST model, where the communication topology is the same as the topology of the network. More precisely, we gave randomized algorithms for triangle finding and listing with round complexity $O(n^{2/3})$ and $O(n^{3/4})$, respectively, where n denotes the number of nodes of the network. We also showed a lower bound $\Omega(n^{1/3})$ on the round complexity of triangle listing, which also holds for the CONGEST-CLIQUE model.

These results have been published in the Proceedings of PODC 2017.

Design of fast quantum distributed algorithm computing the diameter

The computation of the diameter is one of the most central problems in distributed computation. In the standard CONGEST model, in which two adjacent nodes can exchange O(log n) bits per round (here n denotes the number of nodes of the network), it is known that exact computation of the diameter requires $\Omega(n)$ rounds, even in networks with constant diameter. We have investigated quantum distributed algorithms for this problem in the quantum CONGEST model, where two adjacent nodes can exchange O(log n) quantum bits per round. Our main result is a $O((nD)^{1/2})$-round quantum distributed algorithm for exact diameter computation, where D denotes the diameter. This shows a **separation between the computational power of quantum and classical algorithms in the CONGEST model**. We also showed an unconditional lower bound $\Omega(n^{1/2})$ on the round complexity of any quantum algorithm computing the diameter, and furthermore show a tight lower bound $\Omega((nD)^{1/2})$ for any distributed quantum algorithm in which each node can use only poly(log n) quantum bits of memory.

These results have been published in the Proceedings of PODC 2018.

Design of fast quantum distributed algorithm for the All-Pairs Shortest Path Problem

The All-Pairs Shortest Path problem (APSP) is one of the most central problems in distributed computation. In the CONGEST-CLIQUE model, in which n nodes communicate with each other over a fully connected network by exchanging messages of O(log n) bits in synchronous rounds, the best known general algorithm for APSP uses $O(n^{1/3})$ rounds. Breaking this barrier is a fundamental challenge in distributed graph algorithms. **We have investigated for the first time quantum distributed algorithms in the CONGEST-CLIQUE model**, where nodes can exchange messages of O(log n) quantum bits, and showed that this barrier can be broken: we constructed a $O(n^{1/4})$-round quantum distributed algorithm for the APSP over directed graphs with polynomial weights in the CONGEST-CLIQUE model. This speedup in the quantum setting contrasts with the case of the standard CONGEST model, for which Elkin et al. (PODC 2014) showed that quantum communication does not offer significant advantages over classical communication.

Our quantum algorithm is based on a relationship discovered by Vassilevska Williams and Williams (JACM 2018) between the APSP and the detection of negative triangles in a graph. The quantum part of our algorithm exploits the framework for quantum distributed search recently developed by Le Gall and Magniez (PODC 2018). Our main technical contribution is a method showing how to implement multiple quantum searches (one for each edge in the graph) in parallel without introducing congestions.

These results have been published in the Proceedings of PODC 2019.

Average-case quantum advantage with shallow circuits

Recently Bravyi, Gosset and König (Science 2018) proved an unconditional separation between the computational powers of small-depth quantum and classical circuits for a relation. **We have showed a similar separation in the average-case setting that gives stronger evidence of the superiority of small-depth quantum computation**: we constructed a computational task that can be solved on all inputs by a quantum circuit of constant depth with bounded-fanin gates (a "shallow" quantum circuit) and showed that any classical circuit with bounded-fanin gates solving this problem on a non-negligible fraction of the inputs must have logarithmic depth. Our results are obtained by introducing a technique to create quantum states exhibiting global quantum correlations from any graph, via a construction that we call the *extended graph*.

These results have been published in the Proceedings of CCC 2019.

Interactive proofs with polynomial-time quantum prover for computing the order of solvable groups

We have considered what can be computed by a user interacting with a potentially malicious server, when the server performs polynomial-time quantum computation but the user can only perform polynomial-time classical (i.e., non-quantum) computation. Understanding the computational power of this model, which corresponds to polynomial-time quantum computation that can be efficiently verified classically, is a well-known open problem in quantum computing. Our result shows that **computing the order of a solvable group, which is one of the most general problems for which quantum computing exhibits an exponential speed-up with respect to classical computing, can be realized in this model**.

These results have been published in the Proceedings of MFCS 2018.

５．主な発表論文等

〔雑誌論文〕　計17件（うち査読付論文　17件／うち国際共著　7件／うちオープンアクセス　17件）

１．著者名 Taisuke Izumi, Francois Le Gall, Frederic Magniez	４．巻 154
２．論文標題 Quantum distributed algorithm for triangle finding in the CONGEST model	５．発行年 2020年
３．雑誌名 Leibniz International Proceedings in Informatics (STACS 2020)	６．最初と最後の頁 23:1-23:13
掲載論文のDOI（デジタルオブジェクト識別子） 10.4230/LIPIcs.STACS.2020.23	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Francois Le Gall	４．巻 137
２．論文標題 Average-case quantum advantage with shallow circuits	５．発行年 2019年
３．雑誌名 Leibniz International Proceedings in Informatics (CCC 2019)	６．最初と最後の頁 21:1-21:20
掲載論文のDOI（デジタルオブジェクト識別子） 10.4230/LIPIcs.CCC.2019.21	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

１．著者名 Taisuke Izumi, Francois Le Gall	４．巻 なし
２．論文標題 Quantum Distributed Algorithm for the All-Pairs Shortest Path Problem in the CONGEST-CLIQUE Model	５．発行年 2019年
３．雑誌名 Proceedings of the ACM Symposium on Principles of Distributed Computing (PODC 2019)	６．最初と最後の頁 84-93
掲載論文のDOI（デジタルオブジェクト識別子） 10.1145/3293611.3331628	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

１．著者名 Hirotada Kobayashi, Francois Le Gall, Harumichi Nishimura	４．巻 48
２．論文標題 Generalized quantum Arthur-Merlin games	５．発行年 2019年
３．雑誌名 SIAM Journal on Computing	６．最初と最後の頁 865-902
掲載論文のDOI（デジタルオブジェクト識別子） 10.1137/17M1160173	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

１．著者名 Akinori Kawachi, Kenichi Kawano, Francois Le Gall, Suguru Tamaki	４．巻 E102-D(3)
２．論文標題 Quantum Query Complexity of Unitary Operator Discrimination	５．発行年 2019年
３．雑誌名 IEICE TRANSACTIONS on Information and Systems	６．最初と最後の頁 483-491
掲載論文のDOI（デジタルオブジェクト識別子） 10.1587/transinf.2018FCP0012	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

１．著者名 Francois Le Gall, Ansis Rosmanis, Harumichi Nishimura	４．巻 126
２．論文標題 Quantum Advantage for the LOCAL Model in Distributed Computing	５．発行年 2019年
３．雑誌名 Leibniz International Proceedings in Informatics (STACS 2019)	６．最初と最後の頁 49:1-49:14
掲載論文のDOI（デジタルオブジェクト識別子） 10.4230/LIPIcs.STACS.2019.49	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Francois Le Gall, Tomoyuki Morimae, Harumichi Nishimura, Yuki Takeuchi	４．巻 117
２．論文標題 Interactive Proofs with Polynomial-Time Quantum Prover for Computing the Order of Solvable Groups	５．発行年 2018年
３．雑誌名 Leibniz International Proceedings in Informatics (MFCS 2018)	６．最初と最後の頁 26:1-26:13
掲載論文のDOI（デジタルオブジェクト識別子） 10.4230/LIPIcs.MFCS.2018.26	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

１．著者名 Francois Le Gall, Frederic Magniez	４．巻 なし
２．論文標題 Sublinear-Time Quantum Computation of the Diameter in CONGEST Networks	５．発行年 2018年
３．雑誌名 Proceedings of the 37th ACM Symposium on Principles of Distributed Computing (PODC 2018)	６．最初と最後の頁 337-346
掲載論文のDOI（デジタルオブジェクト識別子） 10.1145/3212734.3212744	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Francois Le Gall and Florent Urrutia	**4．巻** なし
2．論文標題 Improved Rectangular Matrix Multiplication using Powers of the Coppersmith-Winograd Tensor	**5．発行年** 2018年
3．雑誌名 Proceedings of the 29th ACM-SIAM Symposium on Discrete Algorithms (SODA 2018)	**6．最初と最後の頁** 1029-1046
掲載論文のDOI（デジタルオブジェクト識別子） 10.1137/1.9781611975031.67	**査読の有無** 有
オープンアクセス オープンアクセスとしている（また、その予定である）	**国際共著** 該当する

1．著者名 Dean Doron, Francois Le Gall and Amnon Ta-Shma	**4．巻** 81
2．論文標題 Probabilistic Logarithmic-Space Algorithms for Laplacian Solvers	**5．発行年** 2017年
3．雑誌名 Leibniz International Proceedings in Informatics (RANDOM 2017)	**6．最初と最後の頁** 41:1-41:20
掲載論文のDOI（デジタルオブジェクト識別子） 10.4230/LIPIcs.APPROX-RANDOM.2017.41	**査読の有無** 有
オープンアクセス オープンアクセスとしている（また、その予定である）	**国際共著** 該当する

1．著者名 Taisuke Izumi and Francois Le Gall	**4．巻** なし
2．論文標題 Triangle Finding and Listing in CONGEST Networks	**5．発行年** 2017年
3．雑誌名 Proceedings of the 36th ACM Symposium on Principles of Distributed Computing (PODC 2017)	**6．最初と最後の頁** 381-389
掲載論文のDOI（デジタルオブジェクト識別子） 10.1145/3087801.3087811	**査読の有無** 有
オープンアクセス オープンアクセスとしている（また、その予定である）	**国際共著** －

1．著者名 Francois Le Gall and Shogo Nakajima	**4．巻** 73
2．論文標題 Multiparty Quantum Communication Complexity of Triangle Finding	**5．発行年** 2018年
3．雑誌名 Leibniz International Proceedings in Informatics (TQC 2017)	**6．最初と最後の頁** 6:1-6:11
掲載論文のDOI（デジタルオブジェクト識別子） 10.4230/LIPIcs.TQC.2017.6	**査読の有無** 有
オープンアクセス オープンアクセスとしている（また、その予定である）	**国際共著** －

１．著者名 Francois Le Gall and Shogo Nakajima	４．巻 79(3)
２．論文標題 Quantum Algorithm for Triangle Finding in Sparse Graphs	５．発行年 2017年
３．雑誌名 Algorithmica	６．最初と最後の頁 941--959
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s00453-016-0267-z	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

１．著者名 Stacey Jeffery, Robin Kothari, Francois Le Gall and Frederic Magniez	４．巻 76(1)
２．論文標題 Improving Quantum Query Complexity of Boolean Matrix Multiplication Using Graph Collision	５．発行年 2016年
３．雑誌名 Algorithmica	６．最初と最後の頁 1-16
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s00453-015-9985-x	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Stacey Jeffery and Francois Le Gall	４．巻 58
２．論文標題 Quantum Communication Complexity of Distributed Set Joins	５．発行年 2016年
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〔図書〕　計0件

〔産業財産権〕

〔その他〕

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