様　式　C－19、F－19－1、Z－19　（共通）

科研費 KAKENHI

科学研究費助成事業　　研究成果報告書

令和　元 年　 6 月 25 日現在

機関番号：１４３０１
研究種目：研究活動スタート支援
研究期間：2017～2018
課題番号：１７Ｈ０６７８７

研究課題名（和文）From Einstein equations to Tensor Networks

研究課題名（英文）From Einstein equations to Tensor Networks

研究代表者
キャプタ　パウエルピオトル（Caputa, Pawel Piotr）

京都大学・基礎物理学研究所・特定研究員

研究者番号：００８０１７４７

交付決定額（研究期間全体）：（直接経費）　　2,100,000円

研究成果の概要（和文）：私は2つの大きな進歩を遂げました。 一つは、CFTにおける経路積分の複雑さと連続的なTensorネットワークの探査、そしてそれらの変形に基づいて、最終的にCFTからの浄化のもつれの最初の計算を導いた。 私の2行目の研究は、量子場理論における計算の複雑さとAdS / CFTにおけるその幾何学的役割の理解でした。 相互作用するCFTにおける幾何学的複雑さの最初の定義をどうにかすることができ、それが重力に関連しているかどうかを示しました。
量子消光のプローブとして回路の複雑さを使うことも提案しました。 これらすべての論文は、名高く非常にインパクトのあるPRLで発表されました。

研究成果の学術的意義や社会的意義
My ground-breaking results opened new unexplored paths for research that uses ideas from quantum information, quantum computation and computational complexity in the framework of continuous quantum field theories. In the future I will use them as tools quantum field theories and genuine AdS/CFT.

研究成果の概要（英文）：In this project, with my collaborators, I have made 2 major breakthroughs. One was based on exploration of path integral complexity and continuous Tensor Networks in CFTs and their deformations that finally led to the first computation of Entanglement of Purification from CFT. My second line of research was on computational complexity in quantum field theory and understanding its geometric role in AdS/CFT. I managed to give a first definition of geometric complexity in interacting CFTs and showed that if is related to the gravity action introduced long ago by Polyakov.
I also proposed to use circuit complexity as a probe of quantum quenches and, in an exactly solvable setup, demonstrated that it indeed can capture (it is sensitive to) non-trivial information about the evolution process like Kibble-Zurek scaling. All these papers were published in prestigious and very-high impact PRL.

研究分野：Theoretical Physics

キーワード：AdS/CFT　Tensor Networks　Entanglement Dynamics　Complexity　Quantum Quench　Holography　Conformal Field Theory　Quantum Gravity

１．研究開始当初の背景

We live in the era of information and as our understanding of nature matures, it can be formulated in terms of information-theoretic language. This trend can be clearly seen in many areas of modern physics but especially in the context of black holes and the holographic AdS/CFT correspondence (AdS/CFT対応). The AdS/CFT correspondence is a map between certain, conformal field theories (共形 場理論, from here: **CFT**) in d dimensions and gravity in one-dimension higher spacetimes with negative curvature (Anti-de Sitter space-time: AdS).

Holography in this context encodes the gravitational degrees of freedom in lower dimensional quantum field theory without gravity. In its most rigorous version, AdS/CFT relates supersymmetric conformal field theories in d spacetime dimensions to theories of quantum gravity (string theory) in d+1 dimensional AdS spacetime.

However, there is a wide belief and evidence that holography holds more generally for strongly interacting systems and can teach us important lessons about this physical regime (present in e.g. quark-gluon plasma or strange metals). However, it is still mysterious what is the basic mechanism between AdS/CFT and its exact range of validity. This is a very active topic of research worldwide.

The main breakthrough in understanding holography that attracted huge attention worldwide was done by Ryu and Takayanagi (RT) that proposed how to compute **entanglement entropy** (エンタングルメントエントロピー, the von Neumann entropy of a reduced density matrix) from geodesics in Anti-de Sitter space.

The second important connection that emerged in the recent studies is the idea that **Tensor Networks** (テンソルネットワーク) used for simulating wave functions of quantum many-body systems at critical points are discretized versions of the AdS/CFT correspondence. Among the supports of this argument was the tensor version of the RT formula for entanglement entropy.

The second development, has a particularly big potential to shed light on the true mechanism behind AdS/CFT and elucidate how holography works in strongly interacting many-body systems. However there is a lot of work needed to make this observation a genuine mechanism of holography and in my project I have been developing necessary tools and tests to probe the relation between tensor networks and AdS/CFT.

２．研究の目的

More precisely, the main purpose of this project was to elucidate how quantum states in CFTs encode holographic geometry and what are the constraints imposed on Tensor Networks in holographic CFTs by the Einstein equations. I pursued my questions in the framework of path integrals and circuit complexity of Tensor Networks in holographic CFTs and AdS/CFT correspondence.

There were four main urgent objectives that my project was designed to improve on:

a) Further developing Path Integral Optimization that I discovered just before the start of this project and exploring its various useful applications in field theories and quantum information measures (e.g. entanglement of purification).

b) Exploring the notion of complexity of the continuous Tensor Networks starting from Path Integral Complexity. Then generalize these findings it in some universal way to conformal field theories and explore their relation to gravity.

c) Continue investigation of the dynamics of quantum information in quantum field theories. This was planned to be done by studying quantum quenches and evolution of entanglement entropy in the protocol of smooth quantum quenches in exactly solvable quantum field theories as well as out-of-time-ordered correlation functions in two-dimensional CFTs with the goal to understand the difference between integrable (Rational) and holographic theories.

d) Finally, I wanted to explore what are the constraints imposed on Tensor Networks in holographic CFTs and study how they are related to Einstein equations in the dual holographic side of the AdS/CFT correspondence.

３．研究の方法

Working at the interface of quantum information and quantum computation as well as quantum field theory and AdS/CFT, my research methodology was to use cutting-edge tools form these fields to explore the nature, limitations and implications of the correspondence between Tensor Networks and holography.

I started by exploring Path Integrals in quantum field theories and their complexity in continuous CFTs and their deformations. This tool was developed by me and my collaborators before the start of the project and, with the procedure of Path Integral Optimization, was the first construction of continuous Tensor Networks in interacting CFTs.

Path Integral Optimization that we defined was done by minimizing the so-called Path Integral Complexity. I further explored this notion and studied complexity of continuum Tensor Networks from the perspective of quantum computation and complexity theory. This allowed me to discover a universal way of defining complexity in 2d CFTs.

Entanglement and out-of-time-ordered correlates are one of the best probes of dynamics of quantum information in holographic setups. I have been following the program of classifying CFTs by the way they “scramble” quantum information and evolve entanglement. In this project I used these tools in particular or orbifold CFT that, by turning parameters, interpolate between Rational and non-rational regime.

Using the recent developments in deformations of conformal field theories by the so-called TT operators, I have studied constraints from gravity (radial Gauss-Codazzi equation) that describe these deformations which bring the theory to finite radial direction (finite cut-off).

４．研究成果

I have managed to make several important breakthroughs in the four objectives of my project.

a) Firstly, my exploration of path integral complexity and continuous Tensor Networks in CFTs and their deformations finally led to the first computation of Entanglement of Purification from conformal field theories [2]. This step was a major advance in the field of quantum information in quantum field theories.

b) My second line of research was on computational complexity in quantum field theory. After further exploring the Path Integral Complexity and also using circuit complexity as a probe of quantum quenches [3], I found a method to give the first definition of geometric complexity in arbitrary interacting CFTs based on the underlying Virasoro symmetry.

More precisely, with Javier Magan, we showed that we can find a cost function for energy-momentum gates in CFTs for which complexity if is related to the gravity action [4]. This work was recently published in Physical Review Letters and is attracting a significant attention in our field.

c) I studied the evolution of entanglement entropy during exactly solvable quantum quench and managed to show that its behavior is consistent with Kibble-Zurek scaling for slow quenches as well as universal features in fast quenches [9]. Moreover, with my collaborators [8], we found a new time evolution of out-of-time-oredered correlates in the orbifold CFTs for non-rational but still not chaotic CFT.

d) Last but not least, in [1], I made a major breakthrough in understanding finite cut-off AdS/CFT. We identified which deformations of holographic CFTs lead to finite cut-off geometries up to six dimensions. We also explained the exact role of the gravity constraint, the Gauss-Codazzi equation in finite cut-off holography as a flow equation.

I am continuing this last very promising development to better understand how our deformations can be understood as a disentangler operator in genuine holographic Tensor Networks.

５．主な発表論文等

1. P. Caputa, S. Datta and V. Shyam "Sphere partition functions and cut-off AdS",
JHEP 1905 (2019) 112
http://arxiv.org/pdf/1902.10893.pdf
DOI: 10.1007/JHEP05(2019)112

2. P. Caputa, M. Miyaji, T. Takayanagi and K. Umemoto, "Holographic Entanglement of Purification from Conformal Field Theories ",
Phys.Rev.Lett. 122, 111601 (2019)
http://arxiv.org/pdf/1902.10893.pdf
DOI: 10.1103/PhysRevLett.122.111601

3. Hugo A. Camargo, P. Caputa, D. Das, M. P. Heller,
R. Jefferson, "Complexity as a novel probe of quantum quenches: universal scalings and purifications",
Phys.Rev.Lett. 122 (2019) no.8, 081601
http://arxiv.org/pdf/1902.10893.pdf
DOI: 10.1103/PhysRevLett.122.081601

4. P. Caputa, J. M. Magan, "Quantum Computation as Gravity",
Phys.Rev.Lett. 122 (2019) no.23, 231302
http://arxiv.org/pdf/1902.10893.pdf
DOI: 10.1103/PhysRevLett.122.231302

5. A.Bhattacharyya, P. Caputa, S. R. Das, N. Kundu, M. Miyaji,
T. Takayanagi, "Path-Integral Complexity for Perturbed CFTs ",
JHEP. 07 086 (2018)
http://arxiv.org/pdf/1902.10893.pdf
DOI: 10.1007/JHEP07(2018)086

6. P. Caputa and S. Hirano, "Airy Function and 4d Quantum Gravity",
JHEP 06 (2018) 106
http://arxiv.org/pdf/1804.00942.pdf
DOI: 10.1007/JHEP06(2018)106

7. P. Caputa, N. Kundu, M. Miyaji, T. Takayanagi and K. Watanabe, ”Liouville Action as Path-Integral Complexity: From Continuous Tensor Networks to AdS/CFT ”,
JHEP 1711 (2017) 097
http://arxiv.org/pdf/1706.07056.pdf
DOI: 10.1007/JHEP11(2017)097

8. P. Caputa, Y. Kusuki, T. Takayanagi and K. Watanabe, ”Out-of-Time-Ordered Correlators in (T2)n/Zn”,
Phys.Rev. D96 (2017) no.4, 046020
http://arxiv.org/pdf/1703.09939.pdf
DOI: 10.1103/PhysRevD.96.046020

9. P. Caputa, S. R. Das, M. Nozaki and A. Tomiya, ”Quantum Quench and Scaling of Entanglement Entropy”, Phys.Lett. B772 (2017) 53-57
http://arxiv.org/pdf/1702.04359.pdf
DOI: 10.1016/j.physletb.2017.06.017

I have presented my work in the following conferences/meetings and colloquia:

1. Title: “From Liouville to Nielsen”.

Presented in:

a) Quantum Information in Quantum Gravity 4, GGI Florence, Italy
Link: https://www.ggi.infn.it/showevent.pl?id=277

b) New Frontiers in String Theory, Kyoto, Japan
Link: http://www2.yukawa.kyoto-u.ac.jp/~nfst2018/index.php

2. Title: “Complexity of Path Integrals in 2d Field Theories”.

Presented in:

a) CERN Colloquium January 2019
Link: https://indico.cern.ch/event/765073/

b) Complexity Workshop 2018, AEI Potsdam, Germany
Link: https://gqfi.aei.mpg.de/node/2

3. Title: “Sphere Partition Functions in Cut-off AdS”.

Presented in:

a) Workshop: TTbar and Other Solvable Deformations of Quantum Field Theories, SCGP, Stony Brook, USA
Link: http://scgp.stonybrook.edu/video_portal/video.php?id=4030

６．研究組織

I have started collaborations with the following researchers:

1. Group: Dr. Michal P. Heller, Dr. Ro Jefferson, Mr. Hugo M. Camargo, Albert Einstein Institute, Golm, Germany

2. Dr. Javier M. Magan, Centro Atomico de Bariloche, Argentina

3. Dr. Shouvik Datta, University of California Los Angeles, USA
4. Prof. Shinji Hirano, WITS University, South Africa
5. Msc. Vasudev Shyam , Perimeter Institute, Canada.
6. Prof. Sumit R. Das, University of Kentucky, USA
7. Dr. Diptarka Das, Indian Institute of Technology, Kanpur, India
8. Dr. Masahiro Nozaki, University of Chicago, USA