様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９　（共通）

科学研究費助成事業　　研究成果報告書

令和　３ 年　 ６ 月　 ７ 日現在

機関番号：３２６１２
研究種目：若手研究
研究期間：2018～2020
課題番号：１８Ｋ１３５０２

研究課題名（和文）Critical properties of quantum melting phase transitions

研究課題名（英文）Critical properties of quantum melting phase transitions

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交付決定額（研究期間全体）：（直接経費）　　2,300,000円

研究成果の概要（和文）：ここ数年で実験室には2次元 4 Heの量子液晶相の実在可能性が提案されている。実験と比べる為に量子固体・液晶臨界点の有効場の理論を使ってくりこみ群(functional renormalization group)理論計算を行った。その結果荷電固定点を発見したので量子固体・液晶臨界点は二次相転移が決定した。

研究成果の学術的意義や社会的意義
Quantum phase transitions at extremely low temperatures determine physical properties at higher temperatures. The detailed understanding of the quantum solid-to-liquid crystal phase transition allows for more control of various helium states, and impact superfluidity and superconductivity research.

研究成果の概要（英文）：Laboratory experiments can deposit helium atoms on graphite substrates with extreme control, and are able to form perfect layers with controllable density, in the low-temperature, quantum regime. These experiments show there may exist a state of planar matter between a solid and a liquid, akin to a liquid crystal. The latter is characterized by preferred directions unlike a featureless liquid. As determining the nature of this state directly is difficult, one may instead obtain information about the transition from the solid to the potentially new, liquid-crystalline state.

To compare with future experimental results, we have studied this quantum phase transition with theoretical methods. In particular, using a duality mapping where lattice vibrations (phonons) are represented as forces (gauge fields), we used the functional renormalization group to determine that this transition should be continuous (second-order). Many avenues for future study have been identified.

研究分野：Physics

キーワード：quantum phase transition　quantum melting　liquid crystals　symmetry breaking　renormalization group

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１．研究開始当初の背景

At zero temperature, different phases of matter are separated by *quantum phase transtions*. While real-world systems always have a finite temperature, their properties are often dominated by the existence of a quantum phase transition in the zero-temperature limit. In this context, phases of helium atoms have been studied in great detail, both in bulk and layered form. Helium atoms can be deposited on graphite substrate under precise control, which lead to the formation of both commensurate and incommensurate layers with highly tunable density. At high density, these layers form a two-dimensional quantum solid, and at low density, the atoms move around freely like in a liquit.
In a recent report, several intermediate phases were identified both in Helium-4 and Helium-3 [Nakamura *et al.* Phys. Rev. B 94 180501(R) (2016)]. One of these phases shows some characteristiscs of the so-called *quantum hexatic* phase, that is a quantum state of matter with the symmetry properties of a hexatic liquid crystal. However, it is experimentally very difficult to directly probe the nature of this intermediate phase, although expensive neutron scattering experiments could perhaps yield some information.
Independent of this experimental effort, A. Beekman and coworkers have developped a general quantum field theory description of quantum liquid crystals as the result of the quantum dislocation-mediated melting of solids. Here a duality mapping represents the dislocation topological defects as charged particles, which interact by exchanging gauge fields that take the role of the phonons. In this sense a liquid crystal is a “stress superconductor”: the transverse stress, that is supported in a solid, decays exponentially quickly in a liquid crystal. This effective field theory, based solely on the symmetries of the respective phases, can be employed to perform calculations about the quantum phase transition between quantum solid and quantum liquid crystal.

２．研究の目的

Instead of studying the intermediate helium phase directly, which is difficult and costly, experimentalists should focus on the quantum phase transition itself, and determine critical properties such as the critical exponents of specific heat. These exponents are universal, insensitive to experimental imperfections, and depend only on the symmetry properties of the phase transition. To be able to identify the nature of the phase transition, the present research aims to calculate the critical exponents of the two-dimensional quantum solid-to-liquid crystal phase transition.

３．研究の方法

Using the effective field theory for quantum liquid crystals, renormalization group calculations were carried out for the simplest case possible: dislocation condensation in the two planar directions, without the further complication of the so-called *glide constraint*, which governs dislocation dynamics. The technical difficulty here is that, in the dual picture, the condensates couple to gauge fields (here: the phonons), and these interactions are notoriously difficult to incorporate in the renormalization group. We used a recent development by G. Fejos and T. Hatsuda, who developped a method to consistently calculate a single condensate to a single gauge field in three dimensions (an ordinary superconductor) [Phys. Rev. D 93, 121701(R) (2016), Phys. Rev. D 96, 056018 (2017)]. In the present research we extend this to two condensates (restoring translation symmetry in the two planar directions) coupled to two gauge fields (phonons), with tunable condensate-condensate coupling. In this formalism it is easy to consider general N-component condensates, where our target situation corresponds to N=2. The general situation allows for comparison with other methods.
A numerical study of the same problem was also envisaged, but due to unforeseen complications even in the analytical calculation, this was not brought to fruition within this project.

４．研究成果

We have succesfully carried out the functional renormalization group calculation for the two N-component condensate coupled to two vector gauge fields situation. The result, the structure of the critical points as function of three parameters: the condensate strength λ, the inter-condensate coupling g, and the gauge field coupling e^2, is pictured in Figure 1. The situation without gauge fields (corresponding to a binary superfluid) is the horizontal plane at $e^2 = 0$, and has two stable fixed points. One fixed point is at g=0, and represents the uncoupled case where both fields condense at the same time. The for the fixed point at g>0, only one field condenses due to condensate-condensate repulsion. The flow diagram indicates that any finite gauge coupling ($e^2 > 0$), the stable fixed points are at a finite, critical value $e^2 = e^{2*}$. Again there are two stable fixed points: the one at g=0 corresponds to both fields condensing. This is the case of the hexatic liquid crystal, where translation symmetry is restored in both directions. At the stable fixed point at g>0, only one field condenses; this represents a smectic liquid crystal where only one field condenses, and translation symmetry is restored in only one direction.
From the renormalization group literature, it is known that these charged fixed points (at $e^2 > 0$) are second-order

phase transitions. We have therefore provided evidence that, in sofar this simplied model characterizes the helium experiments, the solid-to-hexatic quantum phase transition is a second-order phase transition. In this simplified model the phase transition is in the U(1)- or 3D*XY*-universality class.

The incorporation of the glide constraint, which states that dislocations can only move in the direction of their Burgers vector, is not expected to alter this conclusion. As it does remove one soft degree of freedom, it may have influence on the corrections to the critical exponent.

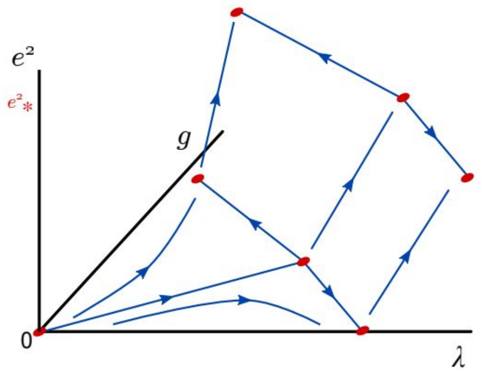

Figure 1: Flow diagram of the two-condensate model as function of condensate strength λ, the inter-condensate coupling *g*, and the gauge field coupling e^2. The existence of charged stable fixed points shows the phase transition is second-order.

During the course of this research project, we have identified several interesting avenues for future investigations. Here are some pointers for both theoretical and experimental researchers:

1. Two parts of the proposed research project were not realized, and should be taken up. First, the effective field theory was simplified in two respects: the glide constraint was omitted, and longitudinal and transverse phonons were taken to have the same velocity. These should be incorporated in the model, and we expect this will not change the second-order nature of the phase transition, but may affect the critical exponents. Similarly, the critical exponents have not been calculated so far, while this information is implicit in the FRG results.
   Second, it is useful to study the same problem numerically, for instance by Monte-Carlo simulations of dislocation worldlines.
2. Experimentalists studying quantum states of matter should also focus their efforts on the critical properties of the quantum phase transition. Due to the fact that the phase transition depends on external parameters like density, while its effects also persist at finite temperatures, there is a wealth of observables available which are all scale-invariant, and hence all have their critical exponents. In turn all these critical exponents are related to each other, and expressed in terms of only two, called η and ν. These quantum critical exponents and their relations are explored by Kirkpatrick and Belitz [Phys. Rev. B 91, 214407 (2015)].
   In our view, some of these exponents should be relatively easy to measure. For the present sitaution, we urge experimentalists to measure the specific heat as function of temperature, right at the critical density between solid and potential-hexatic phases.
3. The effective field theory of quantum liquid crystals that was used in this project has a peculiar property: a quantum solid that loses it transverse rigidiy, i.e. the quatum liquid-crystalline phase, is automatically a superfluid, or a superconductor in the charged case. Indeed one can explictly calculate the Meissner effect. The surprising thing is that this occurs even without the condensation of a bosonic particle field. The reason is that the conservation of particle number, which is the symmetry that is broken in the superfluid, is present in the dislocation condensation via the glide constraint.
   This is contrast to several other studies, that argue that quantum melting and superfluidity are two separate issues. As such, it would be possible to have two separate phase transitions, and a quantum hexatic phase without superfluidity may exist. It is indeed possible that our effective field theory only describes one limit of the full phase diagram; and this should be related to the fact that we do not allow for disclination excitations, which are on a deep level also related to particle conservation. This issue should be researched further. Experimentalists should try to see whether there are two seperate phase transtions, corresponding to hexatic phases with or without superfluidity.
4. Although the quantum phase transition in two dimensions has a three-dimensional universality class, it is also interesting to study the two-dimensional universality. Here, even the ordinary superconductor (one condensate coupled to one gauge field) may show some surprises. In the neutral case, without gauge fields, the Mermin-Wagner-Hohenberg-Coleman theorem prevents long-range order in two dimensions, and one has the Berezinskii-Kosterlitz-Thouless phase transition to a state with algebraic long-range order. This is due to the interplay between vortex interaction and vortex entropy, both of which depend logarithmically on system size.
   However, with gauge fields, there is no soft mode and no infrared divergence which prevents a true long-range ordered state in two-dimensions. On the other hand vortex-vortex interactions are short-ranged, so that entropy always dominates for large systems, and the phase transition is pushed to lower and lower temperatures for smaller sizes. So the paradoxical situation is that, naively, even though long-range order is not prohibited, it does not occur at finite temperatures.
   In real-world two-dimensional superconductors, this whole debate is circumvented as the gauge fields (electromagnetic fields) actually exist in the the third dimension as well, which causes the effective London penetration depth do diverge. Therefore, two-dimensional superconductors behave as two-dimensional superfluids, with a BKT transition, for its non-electronic properties.
   The question what happens in a true two-dimensional superconductor, with two-dimensional gauge fields, remains open. Will the critical point indeed be pushed to zero temperature, or does long-range order exists at some low temperature range? It is worthwhile to study this with renormalization group methods as well as numerical simulations. Experimentally, the situation could perhaps be researched by supressing gauge fields in the third dimension, for instance by shielding the planar sample by short-coherence length superconductors. Perhaps even analogous systems like one-dimensional quantum liquid crystals can be used to study this universality class.

５．主な発表論文等

〔雑誌論文〕　計4件（うち査読付論文　4件／うち国際共著　4件／うちオープンアクセス　4件）

１．著者名 A.J. Beekman and G. Fejos	４．巻 100
２．論文標題 Charged and neutral fixed points in the O(N)+O(N) model with Abelian gauge fields	５．発行年 2019年
３．雑誌名 Physical Review D	６．最初と最後の頁 16005
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevD.100.016005	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 A.J. Beekman, L. Rademaker and J. van Wezel	４．巻 11
２．論文標題 An introduction to spontaneous symmetry breaking	５．発行年 2019年
３．雑誌名 SciPost Physics Lecture Notes	６．最初と最後の頁 1
掲載論文のDOI（デジタルオブジェクト識別子） 10.21468/SciPostPhysLectNotes.11	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

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２．論文標題 Stability and absence of a tower of states in ferrimagnets	５．発行年 2020年
３．雑誌名 Physical Review Research	６．最初と最後の頁 13304
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevResearch.2.013304	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Beekman Aron J	４．巻 7
２．論文標題 Theory of generalized Josephson effects	５．発行年 2020年
３．雑誌名 Progress of Theoretical and Experimental Physics	６．最初と最後の頁 073B09
掲載論文のDOI（デジタルオブジェクト識別子） 10.1093/ptep/ptaa088	査読の有無 有
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〔学会発表〕　計6件（うち招待講演　2件／うち国際学会　4件）

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２．発表標題 The critical point of quantum topological melting
３．学会等名 International Conference on Topological Materials Science 2019（国際学会）
４．発表年 2019年

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２．発表標題 Generalized Josephson effects
３．学会等名 75th Annual Meeting of the Physical Society of Japan
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２．発表標題 The solid-to-hexatic quantum melting phase transition
３．学会等名 International Symposium on Quantum Fluids and Solids 2018（招待講演）（国際学会）
４．発表年 2018年

１．発表者名 A.J. Beekman
２．発表標題 Dislocation-mediated quantum melting
３．学会等名 International Conference on Magnetism 2018（国際学会）
４．発表年 2018年

１．発表者名 A.J. Beekman
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３．学会等名 Workshop on Recent Developments in Chiral Matter and Topology（招待講演）（国際学会）
４．発表年 2018年

１．発表者名 A.J. Beekman, G. Fejos
２．発表標題 Functional renormalization group study of charged two-field theory
３．学会等名 74th Annual Meeting of the Physical Society of Japan
４．発表年 2019年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

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６．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考

７．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計0件

８．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関