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研究種目：基盤研究(B) (一般)

研究期間：2014～2018

課題番号：26287010

研究課題名(和文) アレクサンドロフ空間のリプシッツ構造・崩壊理論とスペクトル逆問題の新展開

研究課題名(英文) New development of Lipschitz structure/collapsing theory of Alexandrov spaces and inverse spectral problem

研究代表者

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研究成果の概要(和文)：1. アレクサンドロフ空間の「良い被覆」という概念を適切に定め、任意のアレクサンドロフ空間が、良い被覆をもつことを示し、それをを用いて非崩壊ケースにおいてリプシッツ・ホモトピー収束定理を得た。2. 断面曲率が下に、境界の第2基本形式が一様に有界であるリーマン多様体が内半径崩壊する場合に、多様体構造を決定した。これはGromov, Alexander-Bishopの結果の拡張を与える。また直径が一様に有界であるとき、余次元1の内半径崩壊の構造を完全に決定した。3. 等周不等式にまつわる測度距離空間の幾何解析や、格子や回転面のスペクトル逆問題についても進展があった。

研究成果の学術的意義や社会的意義

アレクサンドロフ空間は、多様体と呼ばれる曲がった空間の崩壊極限として現れる重要な、特異点をもつ空間である。我々の良い被覆を用いたアレクサンドロフ空間のリプシッツ・ホモトピーの研究は、今まで難解だったアレクサンドロフ空間の研究に、新しい手法を提供する画期的なものである。また我々の境界つき多様体の内半径崩壊の研究は、これまでほとんど知られていなかった境界つき多様体の崩壊の研究の可能性を大きく開く画期的なものといえる。

研究成果の概要(英文)：1. We properly defined the notion of good coverings of Alexandrov spaces, and obtained a Lipschitz homotopy convergence theorem in the non-collapsing case using it. 2. In the case when a manifold with boundary inradius collapses under a lower sectional curvature bound and a two-side bounds on the second fundamental form of the boundary, we determined the manifold structure. This gives an extension of a result due to Gromov, Alexander-Bishop. We also determined the structure of inradius collapse of codimension one in the case of bounded diameter. 3. We developed geometric analysis of metric measure spaces concerning isometric inequalities and spectral inverse problems about lattices and surfaces of revolution.

研究分野：微分幾何学

キーワード：アレクサンドロフ空間 良い被覆 リプシッツ・ホモトピー 内半径崩壊 測度距離空間 スペクトル逆問題

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