

科学研究費助成事業 研究成果報告書

平成 29 年 6 月 9 日現在

機関番号：15201

研究種目：研究活動スタート支援

研究期間：2015～2016

課題番号：15H06416

研究課題名(和文) Thermodynamic formalism for conformal semigroup actions

研究課題名(英文) Thermodynamic formalism for conformal semigroup actions

研究代表者

イエーリッシュ ヨハネス (Jaerisch, Johannes)

島根大学・総合理工学研究科・講師

研究者番号：90741869

交付決定額(研究期間全体)：(直接経費) 2,100,000円

研究成果の概要(和文)：私は、等角半群作用の力学系的性質と幾何学的性質の相互作用を中心に研究しました。主な研究結果の内の2つは以下の通りです。

- 1) 私はマルチフラクタル解析をランダム複素力学系の研究に応用しました。とくに、ランダム複素力学系における初期値の長時間挙動のHoelder条件の指数を研究しました(J. Jaerisch, H. Sumi, Adv. Math. 2017)。
- 2) 私は抽象的な自由群とその正規部分群のPoincare指数を研究しました。その証明においてはCayley graphの離散ラプラシアンを用いました(J. Jaerisch, K. Matsuzaki, Proc. AMS. to appear)。

研究成果の概要(英文)：We have investigated the interplay of dynamics and geometry of conformal semigroup actions. Two of the main results are the following.

- 1) We have applied the multifractal analysis to random complex dynamical systems. In particular, we have investigated the Hoelder exponent of the long-term behavior depending on the initial value (J. Jaerisch, H. Sumi, Adv. Math 313 (2017), 36 pages).
- 2) We have investigated the Poincare exponent of abstract free groups and its normal subgroups. In the proof we use the discrete Laplacian on the associated Cayley graph (J. Jaerisch, K. Matsuzaki, Proc. AMS, to appear 2017).

研究分野：Ergodic theory and dynamical systems

キーワード：Ergodic theory Dynamical systems Fractal Geometry Hyperbolic Geometry Geometric group theory

1 . 研究開始当初の背景

Ergodic theory and thermodynamic formalism can be used to investigate complex behavior in dynamical systems. Moreover, these theories provide fruitful tools to study the interplay between geometric, probabilistic and dynamical aspects. For uniformly hyperbolic dynamical systems, these tools have been successfully applied, and many interesting connections between complex analysis, geometric group theory, fractal geometry and number theory are known.

However, for the systems and questions we consider, either a suitable ergodic theory (in particular, thermodynamic formalism) is not yet available, or it is unknown how to deal with the problem in an ergodic-theoretic framework.

Let us outline some open problems more specifically. H. Sumi (Cooperation principle, stability and bifurcation in random complex dynamics, *Adv. Math.* 245 (2013) 137-181) introduced a class of functions (some of which called complex analogues of the classical Takagi function) associated with random complex dynamical systems, which reflect the long-term behavior of the random dynamical systems, and hence, carry aspects of chaos and order. The modulus of continuity of these functions was not well understood before the research of the project at hand was carried out, although first results indicated that the regularity of these functions is an interesting quantity. This is an example of the lack of a systematic theory to explain chaos and order in random dynamics, or non-autonomous dynamical systems.

Another open problem is to develop the thermodynamic formalism for dynamical systems with a non-compact state space or systems with a lack of recurrence or hyperbolicity. Interesting examples are given by group actions of normal subgroups of free groups (e.g., free Kleinian groups). In this project, we considered critical exponents of normal subgroups of free groups, and the fractal analysis of the geodesic flows on hyperbolic manifolds with cusps.

2 . 研究の目的

One of the main goals of this project is to elaborate the connection between the regularity of the complex analogues of the Takagi function and the thermodynamic formalism and multifractal formalism in

ergodic theory.

Regarding the critical exponents of discrete groups acting on trees, the aim is to understand the interplay between growth and cogrowth of groups. Moreover, to use some of the geometric methods for hyperbolic manifolds also in the abstract setting of groups acting on trees. Here, the concept of isoperimetric constants is known, but the abstract theory is not sufficiently well developed.

Concerning the fractal analysis of geodesic flows on hyperbolic manifolds with cusps, the aim was to establish analogous results as for the modular surface (see J. Jaerisch, M. Kesseböhmer, *The arithmetic-geometric scaling spectrum for continued fractions* Arkiv foer Matematik 48 (2010)), in which arithmetic and geometric aspects have been explored by investigating the continued fraction expansions of numbers.

3 . 研究の方法

The main toolbox is the thermodynamic formalism in ergodic theory. To make the formalism applicable to the complex analogues of the Takagi function, we carefully investigate the interplay between the local behavior of the functions and the underlying dynamical system.

To obtain new results on critical exponents of normal subgroups of free groups acting on trees, we further develop the concept of isoperimetric constants (motivated by corresponding results for hyperbolic manifolds). This allows us to obtain estimates on the spectrum of an associated discrete Laplace operator (on the graph).

4 . 研究成果

The main results are the following.

1) We have shown that the multifractal formalism is applicable to study the pointwise Hölder exponents of the complex analogues of the Takagi function. In this way, we obtained detailed information about the modulus of continuity of these functions. These results have been published in **Adv. Math.** **313** (2017) 839-874, joint with H. Sumi.

2) New estimates and relations between growth and cogrowth of groups acting on trees using the concept of isoperimetric constants have been obtained. These results will appear in **Proc. Amer. Math Soc.**, to appear 2017, joint with K.

Matsuzaki.

3) New results for critical exponents of discrete groups on Gromov hyperbolic spaces (Preprint, joint with Y. Yabuki and K. Matsuzaki).

4) In a joint work with Marc Kesseböhmer and Sara Munday, we performed a multifractal analysis of the cusp winding process for the geodesic flow on certain hyperbolic surfaces (preprint).

5) The revision of our paper on infinitely generated hyperbolic semigroups of rational maps has been completed, and the paper is published in **Trans. Amer. Math. Soc.** **369** (2017), no. 9, 6147-6187 (joint with H. Sumi).

6) The revision of our paper on skew product dynamics of hyperbolic rational semigroups satisfying the separating has been completed. The results are published in **Nonlinearity** **28** (2015) 2913-2938, joint with H. Sumi.

7) Finally, first results have been obtained to generalize a classical amenability criterion of Kesten from groups to graphs (joint with M. Stadlbauer).

5. 主な発表論文等

(研究代表者、研究分担者及び連携研究者には下線)

[雑誌論文](計 5 件)

- 1) J. Jaerisch, K. Matsuzaki, *Growth and cogrowth of normal subgroups of a free group*, **Proc. Amer. Math. Soc.**, to appear.
- 2) J. Jaerisch, H. Sumi, *Dynamics of infinitely generated nicely expanding rational semigroups and the inducing method*, **Trans. Amer. Math. Soc.**, **369** (2017), no. 9, 6147-6187.
- 3) J. Jaerisch, H. Sumi, *Pointwise Hölder exponents of the complex analogues of the Takagi function in random complex dynamics*, **Adv. Math.** **313** (2017), 839-874.
- 4) J. Jaerisch, H. Sumi, *Multifractal formalism for expanding rational semigroups and random complex dynamical systems*, **Nonlinearity** **28** (2015) 2913-2938.
- 5) J. Jaerisch, H. Sumi, *Hölder regularity of limit state functions*

in random complex dynamical systems, **RIMS Kokyuroku** **1942** (2015), 92-98.

[学会発表](計 13 件)

Conference presentations. The presenter is indicated with (*).

- 1) J. Jaerisch (*), K. Matsuzaki, *Growth and cogrowth of normal subgroups of a free group*, MSJ Spring Meeting 2017 at Tokyo Metropolitan University, Japan, 2017.03.24.
- 2) J. Jaerisch (*), H. Sumi, *Multifractal analysis of the random iteration of rational maps on the Riemann sphere*, Conference on Complex dynamics and its applications, invited talk, Kyoto University, Japan, 2016.12.13.
- 3) J. Jaerisch (*), M. Kesseböhmer, S. Munday, *Cusp winding process for some hyperbolic surfaces with cusps*, Autumn Meeting 2016 of the Mathematical Society Japan, research talk in the Probability theory section, Kansai University, Japan, 2016.09.16.
- 4) J. Jaerisch (*), H. Sumi, *Average behaviour of the random iteration of rational maps on the Riemann sphere*, Conference on Dynamical systems and its applications, Kyoto University, Japan, 2016.06.10.
- 5) J. Jaerisch (*), H. Sumi, *Average behaviour of the random iteration of rational maps on the Riemann sphere*, International workshop on "Dynamics, Ergodic Theory and Fractals", invited talk, Osaka University, Japan, 2016.05.21.
- 6) J. Jaerisch (*), H. Sumi, *Hausdorff dimension of the Julia sets of non-hyperbolic polynomial semigroups and the method of inducing*, **Annual meeting of the Mathematical Society Japan, Invited talk in complex analysis session**, Tsukuba University, Japan, 2016.03.16.
- 7) J. Jaerisch (*), K. Matsuzaki, *Critical exponents of normal subgroups of a Kleinian group*, Annual meeting on Riemann surfaces and discontinuous groups, invited talk, Tokyo Institute of Technology, Tokyo, Japan, 2016.02.15

- 8) *J. Jaerisch* (*), *H. Sumi*, Bowen's formula for the Julia sets of some non-hyperbolic polynomial semigroups, RIMS Workshop on Complex dynamics, Kyoto University, Japan, 2015.12.07.
- 9) *J. Jaerisch* (*), *Thermodynamic formalism for infinite group extensions of topological Markov chains*, Annual meeting on ergodic theory, Keio University, Japan, 2015.11.15.
- 10) *J. Jaerisch* (*), *H. Sumi*, On the level of continuity of the complex analogues of the Takagi function, International Workshop on Mathematical Sciences, invited talk, Dalian University, China, 2015.10.30.
- 11) *J. Jaerisch* (*), *H. Sumi*, Hölder regularity of the complex analogues of the Takagi function, AMS Special Session, Fractal Geometry and Dynamical Systems, invited talk, University of Memphis, USA, 2015.10.17.
- 12) *J. Jaerisch* (*), *H. Sumi*, Hölder regularity of the complex analogues of the Takagi function, Autumn meeting of the Mathematical Society Japan, complex analysis session, Kyoto Sangyou University, Japan, 2015.09.13.
- 13) *J. Jaerisch* (*), *H. Sumi*, Hölder regularity of the complex analogues of the Takagi function, Dynamical Systems and its applications, Kyoto University, Japan, 2015.06.30.

〔その他〕

ホームページ等

<http://www.math.shimane-u.ac.jp/~jaerisch>

6 . 研究組織

(1)研究代表者

イエーリッシュ ヨハネス

(JAERISCH, Johannes)

島根大学・総合理工学研究科(研究院)・講師

研究者番号：90741869

(2)研究分担者

()

研究者番号：

(3)連携研究者

()

研究者番号：

(4)研究協力者

角 広輝 (SUMI, Hiroki)

京都大学大学院人間・環境学研究科共生人間学専攻数理科学講座 教授

研究者番号：40313324

松崎 克彦 (MATSUZAKI, Katsuhiko) 早稲田大学, 教育・総合科学学術院, 教授

研究者番号：80222298

KESSEBÖHMER, Marc,

University Bremen, Germany

STADLBAUER, Manuel,

University of Rio de Janeiro, Brasil

MUNDAY, Sara,

University of Bologna, Italy