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Geometric Iwasawa Theory

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positive characteristic

Kato-Trihan　On the conjecture of Birch and Swinnerton Dyer in

Z_p

Z_p Abel   p   Lie   Abel

Abel

The Iwasawa theory for abelian variety over the number fields was initiated in the 70s by Mazur. We give after Kato-Urban-Skinner the second case where the conjecture holds.

We have established a foundation work generalizing the previous work of Kato -Trihan (this is now a submitted paper of more than 200 pages), proved the Iwasawa Main conjecture in two important cases : semistable abelian varieties over the arithmetic extension and constant ordinary abelian varieties over any Z_p
extensions ramifying at a finite set of places.

Mathematics

Iwasawa　Elliptic curve　Function fields

1   The Iwasawa conjecture for the trivial motive for function fields of characteristic p>0 was studied already in 87 by R. Crew. The first time that the analogue of this theory was considered for abelian varieties was by me and Ochiai in 2009. There we proved that the Selmer module over the arithmetic extension was torsion as predicted by Iwasawa. The purpose of this grant was to develop therefore the first steps toward the Iwasawa Main conjecture for such coefficients.

                    We want to develop a geometric analogue of the Iwasawa Theory over number field in the context of function fields of characteristic p>0.

                    Our method is two-folds: without hypothesis on the reduction of the abelian variety, our approach relies on the previous work of myself and Kato, that is, on the resolution of the Birch and Swinnerton-Dyer conjecture by crystalline methods. One of the first goal of our study was therefore to generalize the methods of Kato -T. In the case of constant ordinary abelian variety the ingredients of our proof of the Iwasawa Main conjecture relies on the work of Crew as well as the tool of normic systems and previous work of Tan.

                    We have established a foundation work generalizing the previous work of Kato -Trihan (this is now a submitted paper of more than 200 pages), proved the Iwasawa Main conjecture in two important cases : semistable abelian varieties over non-commutative extensions containing   the arithmetic extension and constant ordinary abelian varieties over any Zp^d extensions ramifying at a finite set of places. Let us develop further these results:

A-Syntomic-Flat comparison Theorem: Me and my co-author Vauclair have established a comparison theorem between flat and crystalline cohomology for semistable abelian varieties over function fields that extends the previous construction of [Kato-Trihan]. More precisely let K denote a function field of dimension 1 over a finite field and consider an abelian variety A/K. Let C denote the geometrically connected projective smooth curve with function field K and A/C the Neron model of A/K. Let Z denote the reduced closed subscheme of C formed by a finite number of points, including all those where A has bad reduction. If U is the open subset of C outside Z, we have the covariant Dieudonne crystal D(A). We have shown
Theorem 1.There exists a canonical Dieudonne crystal D(A) on the log curve (C,Z) extending D(A) such that the mapping fiber of 1-Frobenius on the crystalline cohomology of D(A) (called syntomic cohomology) can be computed by the flat cohomology of the p-torsion points of A.
This approach has been initially used in Kato-Trihan where it is the main ingredient for the proof of the BSD formula under the rank hypothesis. We establish a sheafied version of the comparison result which is more tractable for the application to Iwasawa Theory.
B-Non-commutative Iwasawa Main Theory: We keep the same notations as in A]. We consider a p-adic Lie extension of the base field K and denote I the associated Iwasawa algebra. Me and David Vauclair have studied the non-commutative Iwasawa theory of A over this non-commutative extension.

Theorem 2: There exists an element L in the Whitehead group of a localization of I interpolating the special values of the Hasse-Weil L-function of A/K twisted at Artin representations and whose image in the relative Grothendieck K-group coincides with the element generated by the Pontrjagyn dual of the Selmer group of A over the non-commutative extension. This result does intensive use of the tools develop to establish the theorem 1, as well as algebraic K-group formalism and the theory of normic system introduced by Vauclair. Theorem 2 has then been used to prove the first case of the ETNC for abelian variety over function fields.

C-Commutative IMC for abelian varieties: Meanwhile, with Lai, Longhi and Tan we have established important cases of the IMC for abelian varieties over abelian extensions.
To summarized, our previous research has covered the most important cases of TNC, ETNC and IMC for abelian varieties over the function field.

                    6   Journal article

1-F. Trihan and D. Vauclair, On the non commutative Iwasawa main conjecture for abelian

varieties over function fields, to appear in Documenta Mathematica.(**Reviewed**)

2-**F. Trihan** and D. Vauclair, Equivariant Tamagawa number conjecture for abelian varieties over global fields of positive characteristic, Proceedings of the American Mathematical Society · October 2018. (**Reviewed**)
DOI: 10.1090/proc/14417

3-King Fai Lai, Ignazio Longhi, Ki-Seng Tan and **Fabien Trihan,**
Pontryagin duality for Iwasawa modules and abelian varieties, Transactions of the AMS,
https://doi.org/10.1090/tran/7016(**Reviewed**)

4-K.F. Lai, I. Longhi, K.-S. Tan , **F. Trihan**, The Iwasawa Main conjecture of constant ordinary abelian varieties over function fields, . Proc. Lond. Math. Soc. (3) 112 (2016), no. 6, 1040–1058. (**Reviewed**)

5-K.F. Lai, I. Longhi, K.-S. Tan , **F. Trihan**, The Iwasawa main conjecture for semistable abelian varieties over function fields, Math. Z. 282 (2016), no. 1-2, 485–510. (**Reviewed**)

6-K.F. Lai, I. Longhi, K.-S. Tan , **F. Trihan**, An example of non-cotorsion Selmer group, PROCEEDINGS OF THE AMERICAN MATHEMATICAL SOCIETY, S 0002-9939(2015)12459-8. (**Reviewed**)

4   conference presentation

1-(2016) On the Geometric Iwasawa theory for abelian variety (University JNU, Harish Chandra Institute, India, University of Strasbourg, Rennes, Tokyo Tech).

2-(2017) On the Equivariant Tamagawa conjecture for abelian variety over function fields (Universidad Catolica de Chile).

3-(2018) On the Equivariant Tamagawa conjecture for abelian variety over function fields (International conference.

4-(2019)University of Taiwan (Feb. 2019); On the Tamagawa Number conjecture after Burns-Kakde and Kim.

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8

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