### 科学研究費助成事業

研究成果報告書

科研費

平成 3 0 年 6 月 1 9 日現在 機関番号: 8 2 6 3 6 研究種目: 若手研究(B) 研究期間: 2016 ~ 2017 課題番号: 1 6 K 2 0 9 4 8 研究課題名 (和文) Curved computer generated holography for 3D displays by development of paraxial solutions 研究課題名 (英文) Curved computer generated holography for 3D displays by development of paraxial solutions 研究代表者 ボワズ ジャキン (Boaz, Jackin) 国立研究開発法人情報通信研究機構・電磁波研究所電磁波応用総合研究室・研究員

交付決定額(研究期間全体):(直接経費) 3,000,000円

研究成果の概要(和文):本研究では、漸近解析を用いた曲率を持つ計算機合成ホログラムの計算法を提案した。計算方法の妥当性はシミュレーションで確認し、光学再生による確認も現在取り組んでいる。また、ベッセル関数を用いた計算法も提案し、従来に比べて高速かつ少ないメモリで計算できることを実証した。 漸近解析を用いた計算法については、16ノードのPCクラスタによる並列計算環境を構築し、数百ギガピクセルのホログラムデータを従来に比べて20倍高速に計算できることを実証した。 この計算環境は、デジタルホログラフィ分野への応用も可能である。今回提案した計算法は、曲率を持つホログラフィックディスプレイへの応用が期待できる。

研究成果の概要(英文):i) Paraxial solutions for curved computer-generated holograms were derived using asymptotic analysis. The results are verified using simulated experiments and fabrication of a real cylindrical hologram is in process. Another analytic solution based on Bessel function was developed. The results showed faster computation and lesser memory requirements and was reported as journal publication ii) The developed solutions required hundreds of giga-pixels to be processed and hence a parallel computing algorithm was developed on a 16-node pc cluster. The developed method could achieve 20-times improvement in computation time compared to conventional methods . iii) A non-parallel version of the developed algorithm was re-used for digital holography imaging

application. The results were published as journal article. The developed methods will be useful for curved holographic display developments.

研究分野: Computer generated holography

キーワード: Curved holography Parallel computation Digital holography Wave propagation Diffraction th eory

#### 1. 研究開始当初の背景

Computer generated holography is considered the  $\mathbf{as}$ most promising candidate to satisfy next generation display requirements. However, the specifications of available display devices do not satisfy holographic requirements (pixel-pitch and pixel-numbers). Due to this the view-angle is very narrow and the display size is too small. The solution we propose is to use a curved display instead of a flat display panel. This allows for large view-angle compared to flat panel, even though the specification of the display device is the same. For this, a curved computer-generated hologram (cylindrical or hemi-spherical) has to be computed first. However, the computation methods for curved computer-generated holograms are still immature. Flat hologram computation methods are well understood, and a lot of optimized algorithms exists. We reported numerical solutions for fast calculation of curved holograms. Our earlier reports were non-paraxial solutions which required huge sampling numbers for computation due to Nyquist sampling conditions and large space-bandwidth. One possibility to solve this issue is by utilizing the paraxial solutions, which is an approximation to non-paraxial solutions. However, such a solution has not been reported yet. In this project we try to develop the non-paraxial solutions and also investigate its potential advantages.

# 2. 研究の目的

Paraxial solutions are approximated forms of non-paraxial solutions. In-order to develop paraxial solutions and understand its potentials the following are aimed to be achieved,

- i) The theoretical derivation of paraxial solutions and asymptotic analysis.
- ii) Develop the numerical method and parallel computation algorithm for fast computation of developed solutions
- iii) Test the computed hologram by optical playback using laser.

# 3. 研究の方法

The non-paraxial solutions are defined in a cylindrical co-ordinate system as shown in Figure.1. They define wave propagation from one cylindrical surface to another cylindrical surface, as shown in Figure.2, where the inner surface is the object and the outer surface is the hologram.



The non-paraxial solutions we developed earlier is the starting point of this analysis. The non-paraxial solutions can be mathematically represented as,

$$p(r,\phi,y) = \sum_{n=-\infty}^{\infty} e^{in\phi} \frac{1}{2\pi} \int_{-\infty}^{\infty} P_n(ak_y) e^{ik_y y} \frac{H_n^{(1)}(k_r r)}{H_n^{(1)}(k_r a)} dk_y$$

Where,

$$P_{n}(a,k_{y}) = \frac{1}{2\pi} \int_{0}^{2\pi} d\phi \int_{-\infty}^{\infty} p(a,\phi,y) e^{-in\phi} e^{-ik_{y}y} dy$$

In the above equation, p(a,phi,y) corresponds to the object and p(r,phi,y) corresponds to the hologram data. The numerical computation of the above equation can be achieved using,

Hologram = FFT<sup>-1</sup>[FFT(Object) x TF]

Where the transfer function (TF) is given by,

$$T(a,k_a,r,k_r) = \frac{H_n^{(1)}(k_rr)}{H_n^{(1)}(k_ra)}$$

Where,  $H_n$  represents Hankel functions of the first kind which corresponds to a diverging outgoing wave representing non-paraxial condition. Starting from the above mentioned non-paraxial solution, the paraxial approximated solutions can be derived as follows. The asymptotic form of the propagation factor  $H_n$  is used to approximate the non-paraxial condition and is given as follows,

$$H_n^{(1)}(x) \sim \sqrt{\frac{2}{\pi x}} e^{i(x - n\pi/2 - \pi/4)}$$

For ease of stationary phase analysis, we convert the equations from cylindrical to spherical co-ordinate system (r,  $\theta$ ,  $\phi$ ). Now, using the above approximation, the complete solution becomes,

$$p(R,\theta,\phi) \approx \frac{ik}{\pi} \sqrt{\frac{1}{2\pi R sin\theta}} \Sigma(-i)^n e^{-i\pi/4} e^{in\phi} P_n(R)$$
  
where,  
$$P_n(R) = \int_{-\infty}^{\infty} \frac{p(a,\theta,\phi)}{e^{iR(k_r sin\theta + k_z cos\theta)} dt}$$

$$P_n(R) \equiv \int_{-\infty}^{\infty} \frac{p(a,\theta,\phi)}{k_r^{\frac{3}{2}} H_n^r(k_r a)} e^{iR(k_r sin\theta + k_z cos\theta)} dk_z$$

The above equation is an integral of the

form,

$$p(R) = \int_{-\infty}^{\infty} f(z) e^{iRg(z)} dz$$

The above equation, when subjected to stationary phase analysis the stationary point kz has been found to be,

$$k_z = k \cos\theta$$

Using the above stationary phase point in the complete solution, the final computation equation becomes,

$$p(R,\theta,\phi) \approx \frac{e^{ikR}}{\pi R} \Sigma(-i)^n e^{-in\phi} \frac{p(a,kcos\theta)}{sin\theta H_n(kasin\theta)}$$

The developed solution was verified using simulation experiments. Young's double slit setup with changing the slit width was simulated. The computed fringes show frequency changes as expected and is shown in Figure.3-4 respectively. This verifies the correctness of the developed algorithm



Cylindrical hologram computed using the above equation is currently in fabrication process. Once the fabrication and testing are completed the results will be published.

Another possible solution, that could effectively reduce the computation method based on 3D Fourier domain analysis was derived

$$f(\theta, y_0) = \frac{i}{4\pi} \int [O_s(\theta, \nu) \otimes_{\theta} h(\theta, \nu)] e^{i2\pi\nu y_0} d\nu$$

Where

$$h(\theta,\nu) = e^{i2\pi R\cos\theta \sqrt{\frac{1}{\lambda^2} - \nu^2}} rect \left[\frac{\theta}{\pi}\right]$$

The numerical computing of  $h(\theta, v)$  occupies huge unwanted bandwidth and consumes lot of computation time. This can be avoided if the analytic solution of  $h(\theta, v)$ can be found out. The analytic solution has been derived using Jacobi-Anger expansion as follows

$$h(\theta, \nu) = \hat{h}(\theta, \alpha) = rect \left[\frac{\theta}{\pi}\right] \Sigma[i^n J_n(\alpha) e^{in\theta}]$$
  
Here  $\alpha = 2\pi R \sqrt{1/\lambda^2 - \nu^2}$  and  $J_n$  is the

Bessel function of first kind. Using the analytical expression in numerical computation allows us to reduce the memory occupancy and computation time drastically. The above results are published [2].

The numerical computing algorithm for the above mentioned theoretical solutions is required to be developed. Since holographic display demands huge data processing in the order of hundreds of giga-pixels, it was decided to develop a parallel computing algorithm. A 16-node PC cluster was chosen for the computation. Each node was equipped with one Tesla 2090 GP-GPU. The parallel computing algorithm was initially very slow since it consumed too much time in data transfer between nodes. The parallel computing algorithm was useless unless the data transfer overhead was taken care of. So, we decided to develop the computation flow in such a way that, there is no need for data transfer between the nodes.

The need for data transfer in a parallel computing system, for the computation of a hologram can be understood from the following Figure.5 and Figure.6.



As seen from the above figure, all the pixels in the object plane are required to calculate just one pixel in the hologram plane. Thus, all the nodes in the cluster needs access to all other nodes for object data. This is the root cause of the slow down and is worse when the number of nodes increases, as shown in the figure right (known as Amdhal's law).

Our solution was to totally avoid the need for data transfer between the nodes by decomposing the computations in both the object and hologram planes. This can be achieved by utilizing the shifting property of the Fourier transform operation as shown in Figure.7 below.



Figure-7

For every decomposed object segment (sub-object) of size N/K x N/K ล sub-hologram of size N x N is computed using the shifting property of Fourier transform as shown in the Figure.7. The above operation is independent and can be done for all the sub-objects in parallel. Each sub-object sub-hologram to computation is done within a single node and hence data transfer requirement is completely avoided. Finally, all the computed sub-holograms from each node combined time sequentially are to reconstruct the complete object.

To verify the efficiency of the method, we compared it with the conventional method for computing hologram on a pc-cluster. The most popular and straight forward conventional method in practice is the Transpose-split (TS) algorithm. The difference in data transfer requirements between the conventional method and proposed method can be understood from the Figure.8 and Figure.9 respectively.



It can be seen that the conventional method requires the time expensive Transpose operations (Figure.8) and one data collection operation at the end. While the proposed method (Figure.9) does not require any data communication after the object data is decomposed. Since we completely decompose both the object and hologram plane, we call this method as the decomposition method.

#### 4. 研究成果

Since an electronic display device to test a curved hologram is not available at the moment, we decided to test the decomposition method on a flat hologram. The object data chosen for the experiments consisted of two depth layers as shown in the Figure 10.



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An optical reconstruction setup was built where an LCOS-SLM with  $1024 \ge 768$ pixels was used to display the computed hologram. The optical reconstructions were captured using a camera. A seen from the figure the optical reconstructions are in good agreement with the object data, which verifies the decomposition method.

To understand the significances of the decomposition method in terms of computation time the calculation was performed on a 16-node pc-cluster. Each node was equipped with one GP-GPU and the all the nodes were connected with Gigabit Ethernet. The Fresnel propagation formula was used to compute the hologram in both decomposition method (proposed) and transpose-split method (conventional). The results are expressed as Figure.11 and Figure.12.





Figure.11 corresponds to 2D decomposition and Figure.12 represents row wise decompositions. From the above figures it can be seen that the proposed decomposition method achieves 20-times speed up in computation compared to the conventional method. These results were published [1].

Digital holographic imaging requires wave propagation computation, for which the algorithms developed in this project were found to be suitable. The non-parallel Fresnel propagation algorithm that was developed in the beginning stages of this project was reused for the digital holographic imaging experiments. The results of the digital holographic experiments were published [3].

In conclusion the following are achieved as a result of this research

- i) Paraxial solutions for curved computer-generated holograms were derived using asymptotic analysis. The results are verified using simulated experiments and fabrication of a real cylindrical hologram is in process.
- ii) Another analytic solution based on Bessel function was developed. The results showed faster computation and lesser memory requirements and was reported as journal publication [2]
- iii) The developed solutions required hundreds of giga-pixels to be processed and hence a parallel computing algorithm was developed on a 16-node pc cluster. The developed method could achieve 20-times improvement in

computation time compared to conventional methods [1].

iv) A non-parallel version of the developed algorithm was re-used for digital holography imaging application. The results were published as journal article [3].

The developed methods will be useful for curved holographic display developments.

- 5. 主な発表論文等
- 〔雑誌論文〕(計3件)
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〔図書〕(計0件)

〔産業財産権〕

○出願状況(計0件)

○取得状況(計0件)

- 6. 研究組織
- (1)研究代表者

ボワズ ジャキン (BOAZ, Jackin) 国立研究開発法人情報通信研究機構 電磁 波研究所電磁波応用総合研究室・研究員 研究者番号:00726325