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研究課題名（和文）An adjoint functors approach to models of cognition

研究課題名（英文）An adjoint functors approach to models of cognition

研究代表者
Ｓｔｅｖｅｎ　Ｐｈｉｌｌｉｐｓ（Phillips, Steven）

国立研究開発法人産業技術総合研究所・情報・人間工学領域・上級主任研究員

研究者番号：９０３４４２０９

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研究成果の概要（和文）：二重処理理論は曖昧に定義されており、想定される特質と必ずしも一致するとは限らないため、議論の余地がある。本研究では、圏論（随伴関手）に基づいて認知の二重処理を理解するアプローチを提案した。実験的研究では、二重処理ルートを引き出す課題を設定して行った。本研究における主な理論的結論は、二重処理のassociative / relational (rule-based)様式が、随伴関手であるシービングと呼ばれる圏論構造に関連付けられることである。この結果は、認知的表象や認知プロセスのモデル化に新しい方法論を提供する。

研究成果の学術的意義や社会的意義
Human cognition involves two different styles of thinking: fast, reflexive and effortless (Type 1) versus slow, reflective and effortful (Type 2). The challenge is to provide a coherent theoretical account of the apparent dual nature of cognition.

研究成果の概要（英文）：Dual-process theories are controversial because they are vaguely defined and don't always align with the supposed distinctions. We proposed a category theory (adjoint functors) approach to dual-process aspects of cognition. A series of experiments were conducted to elicit dual-processing routes within a given task. Our main theoretical result was to show that an associative/relational (rule-based) form of dual-process is related by a category (sheaf) theory construction, called sheaving, which is an adjoint functor. This result provides a new way of modeling cognitive representations and processes.

研究分野：Cognitive Science

キーワード：圏論　普遍的構造　システム性　学習　刺激－反応

１．研究開始当初の背景

Human cognition involves two different styles of thinking: fast, reflexive and effortless on one hand versus slow, reflective and effortful on the other. The two styles are called Type 1 and Type 2 by *dual-process* theories of cognition. Dual-process theories are controversial because they are vaguely defined and don't always align with the distinctions. The challenge is to provide a coherent theoretical account of the apparent dual nature of cognition.

２．研究の目的

We propose that dual-process features of cognition are related by adjoint functors: a pair of structure-preserving maps that cohere in a particular way. The project is two-fold: empirical and theoretical. On the empirical side, we conducted a series of experiments, motivated by our category theory approach to cognition (Phillips, Takeda, & Sugimoto, 2016, 2017). Here, we focus on the experimental work designed to test the associative/relational (Type 1/Type 2) distinction and its empirical implications (Phillips, et al, 2016). (Further theoretical work is needed to model the forms of dual process cognition observed in our other experiments so are not examined here.) The empirical implication specifically addressed was the capacity for generalization to novel stimuli – relational (rule-based) systems afford a form of generalization that ssociative systems do not.

On the theoretical side, we proposed a category theory (adjoint functors) account of such dual-process distinctions. Category theory is a branch of (meta)mathematics invented to formalize connections between mathematical constructions. Adjoint functors are essential to establishing such connections. Here, we model the connection between dual-process aspects of cognition using adjoint functors.

We briefly describe one of our experiments as a concrete example that motivated the specific use of adjoint functors that are described in the Methods section. The experiment involved cue-target learning. The cues were pairs of letters and the targets were coloured shapes. Each map from cues to targets was composed of a product of letter-colour and letter-shape maps. Thus, participants could learn a letter pair to coloured shape map either associatively (i.e. without regard to the constituent letters, colours and shapes), or relationally by recognizing the component letter-colour and letter-shape rules (see Fig. 1). Relational process afforded correct prediction on novel cues (letter pairs). Associative processes treat each cue-target instance as unique – no generalization to novel cues is afforded. Response errors revealed that participants employed both associative and relational processing, depending on the number letter pairs to coloured shapes that had to be learned in each task, and the order in which that tasks were conducted (Fig. 2).

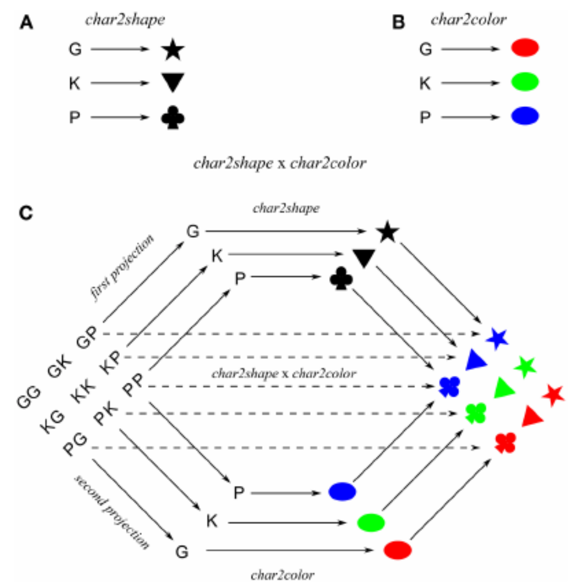

FIGURE 1 | Maps (A) *char2shape* and (B) *char2color* and their product (C) *char2shape* x *char2color*. The dashed arrows direct computation of the product map, solid arrows indicate indirect computation of the product map via the two component maps.

Figure 1: An example map from pairs of letters to coloured shapes (Phillips et al, 2016).

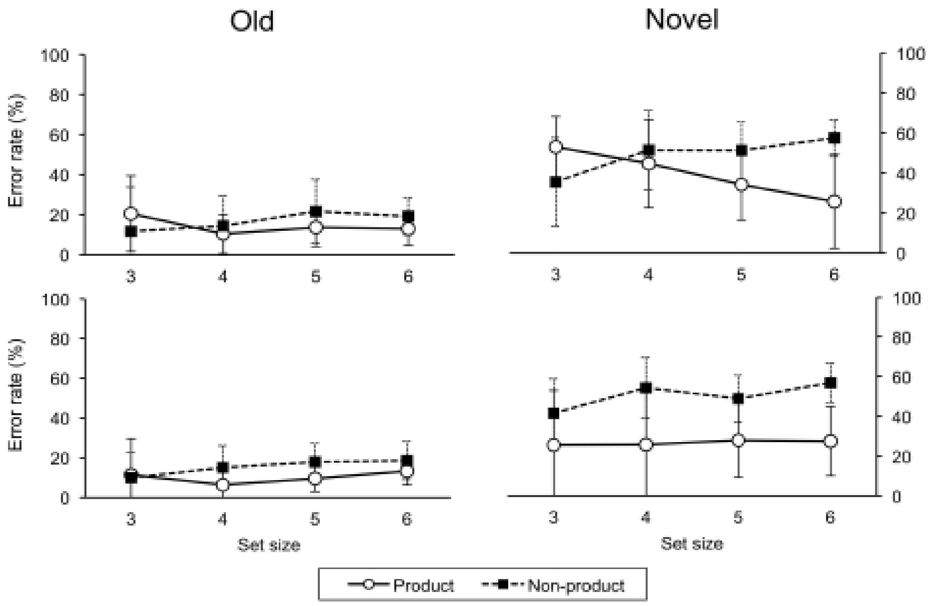

Figure 2: Error rates for training (Old) and testing (Novel) letter pairs for product and non-product cue-target maps (Phillips et al, 2016).

３．研究の方法

This form of dual process is modeled using category (sheaf) theory methods, which involve *sheaves* and *presheaves*. A sheaf or presheaf is a functor, which we use to model cues and targets as data (visual features) attached to a topological space (task dimensions). For intuition, a presheaf/sheaf can be regarded as a relational database table, where the names that make up the table headings correspond to the topological space and the rows (data) of the table correspond to features, such as specific letters, colours, or shapes. See Fig 3 for a comparison of category/sheaf theory concepts with more familiar set theory and relational database theory concepts.

Set theory	**Relational database theory**
Element, set	Column name, header
(assignment) function	(data) table
(higher-order) function	(table) transformation
optimal function	natural join, renormalization
Category theory	**Sheaf theory**
Object/morphism, category	Open set/inclusion, topology
(contravariant) functor	presheaf/sheaf
natural transformation	presheaf/sheaf morphism
universal morphism	pullback, sheaving

Figure 3: Corresponding concepts in set/relational theory and category/sheaf theory (Phillips, 2018).

An example of modeling cues/targets as presheaves and sheaves follows. Suppose cues (letter pairs) GA, GE and KA. The positions of each letter correspond to the points of the underlying topological space and the pairs correspond to the rows of the table shown in Fig 4 (top). A sheaf is a kind of "complete" presheaf: e.g., the sheaf in Fig 4 (bottom) includes the pair KE, which is missing in the presheaf. The crucial sheaf theory construction is called *sheaving*, which takes every presheaf to the nearest sheaf. In this example that means "completing" the presheaf by adding the missing pair, KE.

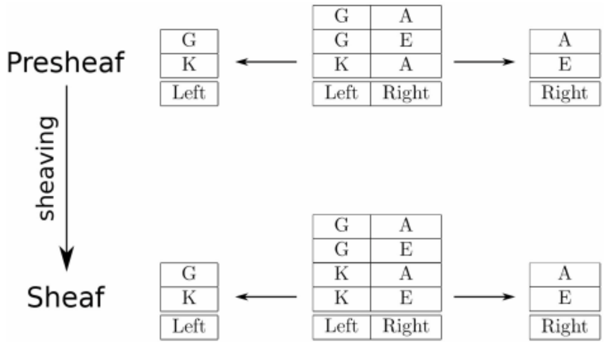

Figure 4: An example of sheaving (Phillips, 2018)

４．研究成果

The main (theoretical) result is to provide a sheaf theory basis for the associative-relational form of dual-process cognition observed in our experiments is modeled as an instance of sheaving (Fig 5). Cues and targets are presheaves/sheaves, hence cue-target maps are presheaf/sheaf morphism. The training set is a subset of the mappings for the given task. Hence, the training set is a presheaf morphism. Generalization to the testing set, containing novel letter pairs and coloured shapes, obtains from the sheaving construction, which realizes all training and testing mappings as the sheaf morphism. Thus, we modeled the relational generalization aspect of this task. The sheaving functor is (left) adjoint to the inclusion functor. Thus, training and testing are formally related by adjoint functors.

This approach also captures the lack of generalization, which is a property of assocative processes, observed in the small set-size conditions by regarding the underlying space as the *indiscrete* topological space. In this case, the two feature dimensions are essentially a single dimension, i.e. each cue/target is treated as a single (chunked) item without regard to its constituent letters, colours and shapes. In this situation, the presheaf corresponding to the training is already a sheaf. So, sheaving returns just the training set – there is no generalization to the testing set. The shape of the space (i.e. indiscrete versus discrete) captures the difference between generalization and no generalization (Phillips, 2018). The sheaves attached to these two spaces are also related by another pair of adjoint functors, which we suggest accounts for associative-relational form of dual-process (Phillips, 2019).

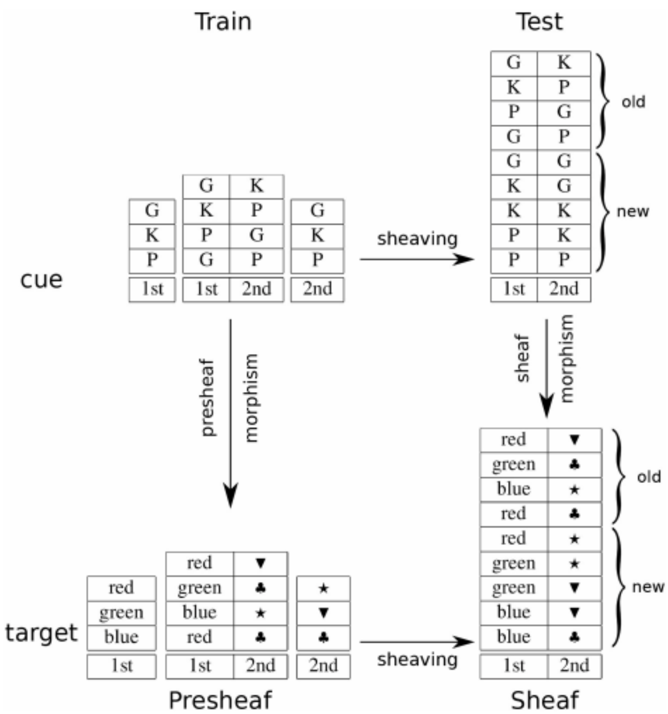

Figure 5: An example of generalization as sheaving (Phillips, 2018).

Additional theoretical work showed that sheaving captures other aspects of cognition, such as visual feature binding (Phillips, 2018b, 2019). Our adjoint functors approach approach opens up a new way to model cognitive representations and processes, which we are continuing to develop.

５．主な発表論文等

〔雑誌論文〕　計8件（うち査読付論文　8件／うち国際共著　3件／うちオープンアクセス　6件）

１．著者名 Phillips, S.	４．巻 9
２．論文標題 Going beyond the data as the patching (sheaving) of local knowledge.	５．発行年 2018年
３．雑誌名 Frontiers in Psychology	６．最初と最後の頁 1926
掲載論文のDOI（デジタルオブジェクト識別子） 10.3389/fpsyg.2018.01926	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 －

１．著者名 Phillips, S., Takeda, Y., & Sugimoto, F.	４．巻 8
２．論文標題 Dual-routes and the cost of determining least-cost.	５．発行年 2017年
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１．著者名 Phillips, S., & Takeda, Y.	４．巻 40
２．論文標題 Mathematical fixation: Search viewed through a cognitive lens.	５．発行年 2017年
３．雑誌名 Behavioral and Brain Sciences	６．最初と最後の頁 40~41
掲載論文のDOI（デジタルオブジェクト識別子） 10.1017/S0140525X16000224, e152	査読の有無 有
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２．論文標題 Systematicity and a categorical theory of cognitive architecture: universal construction in context.	５．発行年 2016年
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2．論文標題 Second-order systematicity of associative learning: a paradox for classical compositionality and a coalgebraic resolution.	5．発行年 2016年
3．雑誌名 PLoS ONE	6．最初と最後の頁 160619(1~26)
掲載論文のDOI（デジタルオブジェクト識別子） 10.1371/journal.pone.016061	査読の有無 有
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1．著者名 Phillips, S., & Wilson W. H.	4．巻 7
2．論文標題 Commentary: Experimental evidence for compositional syntax in bird calls.	5．発行年 2016年
3．雑誌名 Frontiers in Psychology	6．最初と最後の頁 1171(1~4)
掲載論文のDOI（デジタルオブジェクト識別子） 10.3389/fpsyg.2016.01171.	査読の有無 有
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2．論文標題 Why are there failures of systematicity? The empirical costs and benefits of inducing universal constructions.	5．発行年 2016年
3．雑誌名 Frontiers in Psychology	6．最初と最後の頁 1310(1~12)
掲載論文のDOI（デジタルオブジェクト識別子） 10.3389/fpsyg.2016.01310	査読の有無 有
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1．著者名 Phillips, S., & Takeda, Y.	4．巻 40
2．論文標題 Mathematical fixation: search viewed through a cognitive lense.(in press)	5．発行年 2017年
3．雑誌名 Behavioral and Brain Sciences	6．最初と最後の頁 40-41
掲載論文のDOI（デジタルオブジェクト識別子） in press	査読の有無 有
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2017年

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2．発表標題
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3．学会等名 Proceedings of the 38th Annual Conference of the Cognitive Science Society（招待講演）（国際学会）
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1．著者名 Phillips, S.	4．発行年 2018年
2．出版社 Cognitive Science Society	5．総ページ数 6
3．書名 Proceedings of the 40th Annual Conference of the Cognitive Science Society	

1．著者名 Phillips, S.	4．発行年 2017年
2．出版社 Springer International Publishing	5．総ページ数 9
3．書名 Artificial General Intelligence (AGI 2017), Lecture Notes in Artificial Intelligence	

1．著者名 Phillips, S.	4．発行年 2017年
2．出版社 Cognitive Science Society	5．総ページ数 1
3．書名 Proceedings of the 39th Annual Conference of the Cognitive Science Society	

〔産業財産権〕

〔その他〕

－

６．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考
研究分担者	武田　裕司 (Takeda Yuji) (10357410)	国立研究開発法人産業技術総合研究所・情報・人間工学領域・研究チーム長 (82626)	