様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９　（共通）

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研究課題名（和文）Expanded bandwidth for production of the optical vortex by cyclotron radiation

研究課題名（英文）Expanded bandwidth for production of the optical vortex by cyclotron radiation

研究代表者

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研究成果の概要（和文）：モデルに依存しない量子ダイナミクス制御の様々な方法の研究に関して４つの論文を発表した。直方導波管内でサイクロトロン運動する荷電粒子の古典輻射減衰の研究について１つの論文の発表を行なった。円筒導波管内で発生する光渦に関して、研究をすでにまとめており、論文として発表する予定である。

上記の４つのモデルに依存しない量子ダイナミクス制御について１次元有効モデルによって調べた。通常とは異なる次数の例外点の存在によって、分数の指数によって特徴付けられる特異な崩壊ダイナミクスを示すことを明らかにした。この量子力学に基づいて解析された結果は、導波管内の古典減衰に対しても同様の振る舞いが見ら

研究成果の学術的意義や社会的意義
The optical vortex has many uses or proposed uses, from optical tweezers to quantum information processing. However a limitation has been that it is difficult to produce outside the optical regime. Our results should provide a step forward to remedying this limitation.

研究成果の概要（英文）：I have published four works that explore various protocols and ideas for quantum dynamical control in a model-independent context. And I have published one work on the classical radiation damping problem of a charged particle undergoing cyclotron motion inside a rectangular waveguide. The final work on the production of the optical vortex in a cylindrical waveguide will be published soon.

Among the four model-independent works, one establishes the dynamics occurring when an emitting system is tuned to the continuum threshold (or cut-off mode in a waveguide) in an effective 1-D system. I showed the presence of an anomalous-order exceptional point that drives the dynamics, resulting in a power-law decay with fractional exponent. This was written in terms of a quantum system but it would apply for the particle undergoing cyclotron motion as well when the frequency is tuned to the cut-off mode in the waveguide.

研究分野：dynamics of open quantum systems

キーワード：optical vortex　non-Markovian dynamics　bound state in continuum　continuum threshold　exceptional point　radiation damping　waveguide

様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９（共通）
Savannah Garmon
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Expanded bandwidth for production of the optical vortex by cyclotron radiation

１．研究開始当初の背景

Recently, much attention has focused on the creation of light with non-zero orbital angular momentum, also known as the **optical vortex**. Whereas in the past we have usually characterized light by frequency, wave number and polarization, the inclusion of non-zero angular momentum introduces a screw-like shape to the optical wave profile, which in turn opens the road to potential new applications in diverse areas such as information processing in the quantum realm and micro-control devices such as optical tweezers.

In the traditional picture of light propagation, the orbital angular momentum variable l is zero $l = 0$. (We emphasize that since l describes the rotation of the wavefront about the central propagation axis, it must be quantized.) In the $l = 0$ case, the Poynting vector, which describes the directional energy flux of the light, aligns with the beam propagation axis such that the light intensity is highest directly on this axis. However, in the case non-zero orbital angular momentum case $l \neq 0$, the Poynting vector instead rotates around the propagation axis with a fixed cycle determined by l, while the beam intensity is zero on the exact propagation axis.

Although production of the optical vortex in the lab was at first considered impractical, the appearance of two papers (one theoretical, one experimental) in 1992 opened the field for the future [1, 2]. More recently, a paper appeared in 2017 which demonstrated that a charged particle experiencing cyclotron motion should radiate light with non-zero angular momentum [3]. This should make it easier to produce the optical vortex at electromagnetic frequencies outside the visible spectrum and enable other applications.

２．研究の目的

The primary goals of this project were to study the radiation damping properties of particle undergoing cyclotronic motion under the influence of an applied magnetic field in a metallic waveguide, and, more generally, to establish characteristics of model-independent decay dynamics that would be applicable to the optical vortex problem under appropriate circumstances. While the ultimate goal for the model-dependent aspect of the problem is a cylindrical waveguide in which the optical vortex is most naturally expected to appear, we also considered a rectangular waveguide.

Within the model-independent portion of the project, one objective is to investigate circumstances in which various forms of both irreversible (Markovian) and reversible (non-Markovian) forms of decay may occur. Particularly, we note that while Markovian decay of the exponential form $P(t) \sim e^{-\Gamma t}$ (in which $P(t)$ is the survival probability of some initially-prepared quantum state) is quite common in simple quantum systems, the origins of non-Markovian dynamics tend to be more complex.

For example, in nuclear decay and the relaxation process of atomic systems, we tend to think of the decay process as essentially exponential. The exponential decay in these cases appears in association with the so-called resonance state, or generalized eigenstate with complex eigenvalue, that appears due to the energy resonance between the microscopic degrees of freedom in the environment and the initially-occupied quantum state (the radioactive nucleus or excited atomic state, for example). However, it is well known that deviations from exponential decay appear must always appear in quantum systems, at least on very short and extremely long time scales [4]. The long-time deviations occur due to existence of a lower threshold on the energy continuum in quantum systems [5], although this is considered to be very difficult to observe in ordinary circumstances because the long-time effect does not appear until after many lifetimes of the exponential decay have passed, by which time little remains of the initial state that can be measured in experiment. One objective of the present study was to understand general circumstances in which the non-exponential (non-Markovian) dynamics could be controlled and enhanced.

Further, we considered circumstances in which the exponential dynamics could be modified, while still remaining Markovian. In this case, we considered the dynamics near a so-called exceptional point, at which two eigenstates coalesce in non-Hermitian systems [6]. Note that an open quantum system, which is the primary model we consider here, can be considered as a minimal example of a non-Hermitian system. Further, in one case we extend the open quantum system to incorporate an explicitly non-Hermitian potential, as discussed below.

３．研究の方法

The methods used in this project were primarily the standard methods used in the analysis of open quantum systems and low-energy physics, although these methods were extended with specialized protocols in some specific cases. As a basic tool of analyzing the spectrum of open quantum systems, a natural starting point is the Green's function or the resolvent operator [7, 8]. If we are interested in the spectral and dynamical properties of the initial state $|q\rangle$, then we write the Green's function as

$$G(z) = \left\langle q \left| \frac{1}{z - H} \right| q \right\rangle$$

in which H is the system Hamiltonian and z is the complex energy variable. In this formalism, the spectrum of the system is equivalent to the poles of the Green's function as one sweeps through the coordinate z.

Next, to analyze the system dynamics, we usually write the survival probability $P(t) = |A(t)|^2$, which is written as the square modulus of the survival amplitude

$$A(t) = \langle q | e^{-iHt} | q \rangle$$

for the initial quantum state $|q\rangle$ (the quantum emitter). For practical calculations, this quantity is generally transformed into an integral over the Green's function, in which z becomes the integration variable. The integration contour generally surrounds the real z axis in the complex energy plane, including the band or continuum dispersion $E(k)$ that describes the energy dispersion associated with the environment. This continuum is mathematically equivalent to a branch cut in the complex energy plane. For practical calculations, we deform the integration contour by dragging it down to negative infinity in the complex energy plane, where most of the integration contribution vanishes. However, this leaves (A) any poles associated with the bound states, (B) any poles associated with a resonance state, and (C) a portion of the contour associated with the branchpoints of the cut. This last contribution gives the long-time decay mentioned previously.

However, for the analysis of the dynamics near an exceptional point in some cases, we find it useful to extend this to a more powerful formalism in which the complex energy states can be treated as true eigenstates, as described in Ref. [9]. This allows us to carefully keep track of the different contributions coming from the pole and the branch cut, which enables us to evaluate the dynamics at the EP in Ref. [10].

Finally, the radiation damping problem itself is technically a *classical* problem, rather than quantum. However, our familiarity with the waveguide problem under a truly quantum scenario [11] provides us the intuition to understand that we can deal with the radiation damping problem for the electron undergoing cyclotron motion in a waveguide in close analogy with the quantum problem; the primary issue is simply that one should replace the quantum commutator brackets with classical Poisson brackets [12].

４．研究成果

The project at first evolved along a somewhat different trajectory than the original plan, with greater focus on the model-independent features of quantum decay in the first phase, with several publications along these lines [10, 13, 14, 15]. However, we returned to the specific objective regarding the radiation damping process of the charged particle under cyclotron motion in the last stretches of the project [12].

In the first stage, we studied a new method to eliminate exponential decay completely so that the long-time power law decay (which is normally extremely difficult to detect in experiment) will dominate the time evolution. We accomplished this by taking advantage of an interesting concept from quantum mechanics called the bound state in continuum (BIC) [16], which can be understood as a resonance state with vanishing decay width. At the exact point at which the decay width vanishes, which occurs due to quantum interference, the remaining real eigenvalue resides directly inside the scattering continuum that is usually associated with delocalized scattering states, although the BIC is compact. We proposed a quite general feature of quantum systems, which is that by initializing the system in a state that resides orthogonal to the BIC, one can suppress the usual exponential decay features in favor of the non-Markovian, non-exponential deviations. We further proposed to measure this in an optical waveguide array experiment [13]. Note that since the BIC can occur in systems outside of the quantum context, this effect could apply in some classical systems as well [16].

Next we analyzed the dynamics of a generic quantum emitter coupled to a 1-dimensional continuum system in Ref. [10]. We demonstrated that due to the Van Hove singularity generically occurring in the density of states in such systems [7], a special type of exceptional point (EP) is expected to generically form when the quantum emitter energy is tuned closely to the band edge or continuum threshold. We showed the general conditions under which this effect is expected to occur and further showed that the EP results in an unusual fractional power law decay for the quantum emitter of the form $P(t) \sim 1 - Ct^{3/2}$ in which C is a constant that depends on the coupling between the emitter and the continuum [10]. Note that this model-independent effect should directly apply to the production of the optical vortex when the cyclotron frequency of the charged particle in that scenario is tuned to the decay channel (in other words, the charged particle under the influence of the external magnetic field would essentially play the role of the "quantum" emitter in this situation).

Generalizing from this work [10] as well as an older paper [17], in the next stage I wrote a review paper on the influence of exceptional points on decay in quantum systems [14]. Again, this work is placed in the model-independent context.

Next, we considered a model consisting of a non-Hermitian potential attached to two semi-infinite leads in Ref. [15]. However, in this case the sites on the leads are dimerized, which introduces non-trivial topological features into the system. Further, the central potential exhibits parity-time (PT) symmetry. We demonstrated circumstances in which this system exhibits a resonance in continuum (RIC), which is essentially a case in which a resonance state becomes embedded in the continuum, but *without* becoming a localized state [15]. This state, which is also known as a spectral singularity in the literature, is a simple example of a lasing state. Note this can only occur in systems that incorporate some non-Hermitian element.

Finally, we turned to the direct objective regarding the dynamics of the charged particle undergoing cyclotron motion in a rectangular waveguide. We have so far evaluated the radiation damping problem for a charged particle in a rectangular waveguide in Ref. [12], in which we illustrated the main point that the dynamics for this classical model can be evaluated largely in analogy with the quantum picture. We further demonstrated that by tuning the cyclotron frequency near or away from the waveguide mode cut-off frequency, one can control the relative intensity of the Markovian and non-Markovian dynamical contributions. Very near the cut-off mode, the dynamics would be largely non-Markovian, similar to the model-independent result in Ref. [10].

The work on the final objective of the charged particle inside a cylindrical waveguide (in which we should be able to see the production of the optical vortex itself) is about half-complete now and should be published within a year. This should allow a significant enhancement in the frequency range at which the optical vortex can be produced. Hence, we will have accomplished most of the main objectives of the original proposal soon, although taking somewhat longer than the original plan.

[**1**] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Phys. Rev. A **45**, 8185 (1992).

[**2**] V. Yu. Bazhenov, M. S. Soskin, and M. V. Vasnetsov, J. Mod. Opt. **39**, 985 (1992).
[**3**] M. Katoh, *et al*, Phys. Rev. Lett. **118** 094801 (2017).
[**4**] L. Fonda, G. C. Ghirardi, and A. Rimini, Rep. Prog. Phys. **41**, 587 (1978).
[**5**] L. A. Khalfin, Sov. Phys. JETP **6**, 1053 (1958).
[**6**] M.-A. Miri and A. Alù, Science **363**, ear7709 (2019).
[**7**] G. D. Mahan, *Many-Particle Physics*, 2^{nd} ed. (Plenum Press, New York, 1990).
[**8**] E. N. Economou, *Green's Functions in Quantum Physics*, 2^{nd} ed. (Springer-Verlag, Berlin, 1983).
[**9**] N. Hatano and G. Ordonez, J. Math. Phys. **55**, 122106 (2014).
[**10**] S. Garmon, G. Ordonez, and N. Hatano, Phys. Rev. Research **3**, 033029 (2021).
[**11**] T. Petrosky, C.-O. Ting, and S. Garmon, Phys. Rev. Lett. **94**, 043601 (2005).
[**12**] Y. Goto, S. Garmon, and T. Petrosky, Prog. Theor. Exp. Phys. **2024**, 033A02 (2024).
[**13**] S. Garmon, K. Noba, G. Ordonez, and D. Segal, Phys. Rev. A **99**, 010102(R) (2019).
[**14**] S. Garmon, T. Sawada, K. Noba, and G. Ordonez, J. Phys.: Conf. Ser. **2038**, 012011 (2021).
[**15**] S. Garmon and K. Noba, Phys. Rev. A **104**, 062215 (2021).
[**16**] C. W. Hsu, B. Zhen, A. D. Stone, J. D. Joannopoulos, and M. Soljacic, Nat. Rev. Mater. **1**, 16048 (2016).
[**17**] S. Garmon and G. Ordonez, J. Math. Phys. **58**, 062101 (2017).

５．主な発表論文等

〔雑誌論文〕　計6件（うち査読付論文　6件／うち国際共著　5件／うちオープンアクセス　3件）

１．著者名 Goto Yuki、Garmon Savannah、Petrosky Tomio	４．巻 2024
２．論文標題 Enhanced Classical Radiation Damping of Electronic Cyclotron Motion in the Vicinity of the Van Hove Singularity in a Waveguide	５．発行年 2024年
３．雑誌名 Progress of Theoretical and Experimental Physics	６．最初と最後の頁 033A02-1--25
掲載論文のDOI（デジタルオブジェクト識別子） 10.1093/ptep/ptae021	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 ―

１．著者名 Dunham Yujin、Kanki Kazuki、Garmon Savannah、Tanaka Satoshi、Ordonez Gonzalo	４．巻 103
２．論文標題 Coherent single-spin electron resonance spectroscopy manifested at an exceptional-point singularity in doped polyacetylene	５．発行年 2021年
３．雑誌名 Physical Review A	６．最初と最後の頁 043513-1--18
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevA.103.043513	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 Garmon Savannah、Ordonez Gonzalo、Hatano Naomichi	４．巻 3
２．論文標題 Anomalous-order exceptional point and non-Markovian Purcell effect at threshold in one-dimensional continuum systems	５．発行年 2021年
３．雑誌名 Physical Review Research	６．最初と最後の頁 033029-1--17
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevResearch.3.033029	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Garmon Savannah、Noba Kenichi	４．巻 104
２．論文標題 Reservoir-assisted symmetry breaking and coalesced zero-energy modes in an open PT-symmetric Su-Schrieffer-Heeger model	５．発行年 2021年
３．雑誌名 Physical Review A	６．最初と最後の頁 062215-1--17
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevA.104.062215	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 Garmon Savannah、Sawada Takafumi、Noba Kenichi、Ordonez Gonzalo	４．巻 2038
２．論文標題 Characteristic influence of exceptional points in quantum dynamics	５．発行年 2021年
３．雑誌名 Journal of Physics: Conference Series	６．最初と最後の頁 012011-1--26
掲載論文のDOI（デジタルオブジェクト識別子） 10.1088/1742-6596/2038/1/012011	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Garmon Savannah、Noba Kenichi、Ordonez Gonzalo、Segal Dvira	４．巻 99
２．論文標題 Non-Markovian dynamics revealed at a bound state in the continuum	５．発行年 2019年
３．雑誌名 Physical Review A	６．最初と最後の頁 010102-1--6
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevA.99.010102	査読の有無 有
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〔学会発表〕　計21件（うち招待講演　6件／うち国際学会　8件）

１．発表者名 Savannah Garmon, Kenichi Noba
２．発表標題 Reservoir-assisted symmetry breaking and coalesced zero-energy modes in an open PT-symmetric SSH model
３．学会等名 Non-Hermitian Quantum Mechanics 2022
４．発表年 2022年～2023年

１．発表者名 Savannah Garmon, Kenichi Noba
２．発表標題 Localization, topology and symmetry-breaking properties of an open PT-symmetric Su-Schrieffer-Heeger model
３．学会等名 Localisation 2022
４．発表年 2022年～2023年

１．発表者名
Savannah Garmon

２．発表標題
Topologically-protected states and their dynamical influence in structured reservoirs with chiral sublattice symmetry

３．学会等名
American Physical Society March Meeting 2023（国際学会）

４．発表年
2022年～2023年

１．発表者名
Savannah Garmon, Kenichi Noba

２．発表標題
Spectral phase diagram and fourth-order exceptional curves in an open PT-symmetric Su-Schrieffer-Heeger model

３．学会等名
Japan Physical Society Autumn Meeting

４．発表年
2021年

１．発表者名
Kenichi Noba, Savannah Garmon

２．発表標題
Reservoir-assisted symmetry breaking in an open PT-symmetric Su-Schrieffer-Heeger model

３．学会等名
Japan Physical Society Autumn Meeting

４．発表年
2021年

１．発表者名
Savannah Garmon

２．発表標題
Non-Markovian dynamics revealed at the bound state in continuum

３．学会等名
Canadian Conference for Undergraduate Women in Physics (CCUWiP)（招待講演）（国際学会）

４．発表年
2022年

１．発表者名 Savannah Garmon, Kenichi Noba
２．発表標題 Reservoir-assisted symmetry breaking and coalesced zero-energy modes in an open PT-symmetric Su-Schrieffer-Heeger model
３．学会等名 American Physical Society March Meeting（国際学会）
４．発表年 2022年

１．発表者名 Savannah Garmon, Gonzalo Ordonez, Naomichi Hatano
２．発表標題 Anomalous exceptional point and non-Markovian Purcell effect at threshold in 1-D continuum systems
３．学会等名 REIMEI Workshop on “Open quantum mechanics in nuclear, hadron and condensed-matter systems”（招待講演）
４．発表年 2022年

１．発表者名 Savannah Garmon
２．発表標題 Anomalous exceptional point and non-Markovian Purcell effect at threshold in 1-D continuum systems
３．学会等名 virtual Pseudo-Hermitian Hamiltonians in Quantum Physics (vPHHQP)（招待講演）
４．発表年 2020年

１．発表者名 Savannah Garmon
２．発表標題 Coexistence of exceptional ring and exceptional surface in a doped molecular chain
３．学会等名 ICTS Non-Hermitian Physics (Online)（招待講演）
４．発表年 2021年

１．発表者名 Savannah Garmon, Yujin Dunham, Kazuki Kanki, Gonzalo Ordonez, Satoshi Tanaka
２．発表標題 Coexistence of exceptional ring and exceptional surface in a doped molecular chain
３．学会等名 American Physical Society March Meeting 2021（国際学会）
４．発表年 2021年

１．発表者名 Savannah Garmon
２．発表標題 Non-Markovian dynamics revealed at the bound state in continuum
３．学会等名 Stat Phys 27 - International Conference on Statistical Physics（国際学会）
４．発表年 2019年

１．発表者名 Savannah Garmon
２．発表標題 Anomalous exceptional point and non-Markovian Purcell effect at threshold in 1-D open quantum systems
３．学会等名 American Physical Society March Meeting 2020（国際学会）
４．発表年 2020年

１．発表者名 Savannah Garmon
２．発表標題 Anomalous exceptional point and non-Markovian Purcell effect at threshold in 1-D continuum systems
３．学会等名 Japan Physical Society 2020 Annual Meeting
４．発表年 2020年

１．発表者名
Savannah Garmon

２．発表標題
Non-Markovian dynamics revealed at the bound state in continuum

３．学会等名
American Physical Society March Meeting 2019（国際学会）

４．発表年
2019年

１．発表者名
Savannah Garmon

２．発表標題
Measuring non-exponential decay at the bound state in continuum

３．学会等名
Japan Physical Society 2019 Autumn Meeting

４．発表年
2018年

１．発表者名
Savannah Garmon

２．発表標題
Measuring non-exponential decay at the bound state in continuum

３．学会等名
EXCON 2018

４．発表年
2018年

１．発表者名
Savannah Garmon

２．発表標題
Measuring non-exponential decay at the bound state in continuum

３．学会等名
Non-Hermitian Physics - PHHQP XVIII（招待講演）

４．発表年
2018年

1．発表者名 Savannah Garmon, Kenichi Noba, Gonzalo Ordonez, Federico Roccati
2．発表標題 Topological exceptional point and spectral properties of a quantum emitter attached to a semi-infinite reservoir with chiral sublattice symmetry
3．学会等名 StatPhys28（国際学会）
4．発表年 2023年～2024年

1．発表者名 Savannah Garmon
2．発表標題 on-exponential dynamics near the continuum threshold: bound states, anti-bound states and exceptional points
3．学会等名 Stat&QuantPhys Winter School 2024 (SQP2024)（招待講演）
4．発表年 2023年～2024年

1．発表者名 Savannah Garmon
2．発表標題 Dynamics of a quantum emitter coupled to a topological structured reservoir
3．学会等名 POS&BYD: Physics of Open Systems and Beyond
4．発表年 2023年～2024年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

－

6．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考

7．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計0件

８．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関			
米国	Butler University	University of Texas		
カナダ	University of Toronto			