様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９　（共通）

科学研究費助成事業　　研究成果報告書

令和　５ 年　 ６ 月 ２２ 日現在

機関番号：３２６８９
研究種目：基盤研究(C)（一般）
研究期間：2018～2022
課題番号：１８Ｋ０４６２１

研究課題名（和文）Non-parametric Bayesian approach to modelling system reliability

研究課題名（英文）Non-parametric Bayesian approach to modelling system reliability

研究代表者
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交付決定額（研究期間全体）：（直接経費）　　2,200,000円

研究成果の概要（和文）：ベイズ的ノンパラメトリックの観点からシステムをモデル化する研究を行った。Lo and Weng (1989) のアプローチを拡張し、Gamma process priorを実装するためにガンマスケールされたDirichlet process priorを使用し、4つのバスタブ型ハザード率関数モデルのシミュレーションを実施したGeometric-like process、故障ハザードに依存する非ゼロ修理時間、Alternating geometric processの平均と分散関数、ワランティにおける故障報告の遅延に関するプロジェクトにも取り組んだ。

研究成果の学術的意義や社会的意義
Based on work done by Lo and Weng (1989), we use Gamma process prior to specify non-parametric hazard rate functions. We have implemented simulation and inference for four bathtub hazard rate models. Our use of the translated Gamma Process prior for the log-convex hazard rate model is novel.

研究成果の概要（英文）：Richard Arnold, Stefanka Chukova and Yu Hayakawa have carried out a work on modelling system from a Bayesian non-parametric perspective. Hazard rate functions of natural and manufactured systems often show a bathtub shaped hazard rate. Parametric modelling of such hazard rate functions can lead to unnecessarily restrictive assumptions on the function shape. We have extended Lo and Weng (1989) approach and specified four non-parametric bathtub hazard rate functions drawn from Gamma Process Priors. We use a gamma-scaled Dirichlet Process prior to construct the Gamma Process Prior, and have implemented simulation and inference for these four models. We and Sarah Marshall also worked on other projects on geometric-like processes, the alternating alpha-series process, nonzero repair times dependent on the failure hazard, mean and variance of an alternating geometric process, and delayed reporting of faults in warranty claims. These projects are complementary to our original goals.

研究分野：Reliability Theory

キーワード：System reliability　Bayesian non-parametrics　Gamma process　Bathtub hazard rate　Warranty analysis　Geometric-like process　Alpha-Series process

１．研究開始当初の背景 (Initial Background of the Study)

Reliability theory is the study of the lifetimes of systems: both manufactured (e.g. cars) and natural (e.g. lengths of time to rock fracture). Models for these situations often concentrate on the hazard rate – the likelihood of imminent failure at any given time t, conditional on survival up to that moment.

The simplest hazard rate functions are monotonic: IFR (non-decreasing hazard rate), DFR (non-increasing hazard rate), and the special intermediate case of the exponential distribution which has a constant hazard rate. Our interest is in the more general set of hazard rate functions that show a bathtub shape: An initial DFR period occurs after manufacture. This is followed by a period of constant – or slowly increasing – hazard rate (CFR), which is the peak productive life of the system. Finally the system begins to deteriorate, and enters and IFR period during which time failure becomes ever more likely.

Parametric models for the hazard rate have frequently been used in studies of reliability. Non-parametric models have also been used, though primarily they have been employed in cases where the focus of the analysis was elsewhere, specifically on covariates.

There have however been strong recent advances in both theory and practice of non-parametric estimation, especially in the Bayesian context. These approaches, based on stochastic processes have enabled practical estimation of the hazard rate function, without specifying the restrictive constraints on its form that are implied by parametric methods.

２．研究の目的 (Purpose of the study)

The purpose of this research project is to extend the class of bathtub hazard rate models using Bayesian non-parametric approaches. This modelling strategy is particularly appealing because it is very flexible, requires few prior assumptions about the hazard rate, and importantly includes the standard parametric models as special cases. Where prior information about the hazard rate function $\lambda(t)$ does exist it can be incorporated through the stochastic processes that are used as prior distributions. More specifically this research project aims to extend the Lo & Weng (1989) approach and specify four non-parametric bathtub hazard rate functions drawn from Gamma Process Priors.

In a system subject to a bathtub hazard rate function, the final IFR period is identifiable with system ageing. However, there are two interpretations of the initial DFR period. One is system-specific: the system is warming up, and so its performance improves during this initial period, with failures becoming less and less likely up to the beginning of the peak performance period. The second is a population-level view: in a population of newly manufactured systems there will be a proportion that are imperfectly constructed and that are likely to fail. As time passes these weaker systems do fail, leaving only the stronger systems which have a lower failure rate. Both points of

view are represented in our proposed research.

Reference:
Lo, A. Y., and Weng, C.-S. (1989). "On a class of Bayesian non-parametric estimates II. Hazard rate estimates," Annals of the Institute of Statistical Mathematics, 41, pp. 221–45.

３．研究の方法 (Methods of the research)

Our models are based on the approach of Lo and Weng (1989) for hazard rate functions. We specify bathtub hazard rate functions which are all based on Gamma Process priors. We use a gamma-scaled Dirichlet Process prior to construct the Gamma Process prior, and implement simulation and inference using a Markov Chain Monte Carlo sampler.

４．研究成果 (Research Results)

(1) Bayesian non-parametric specification of bathtub shaped hazard rate functions

We consider models which are extensions of Lo and Weng (1989) models which can be constructed as a mixture of the kernel. We specify six models using draws from one or more Gamma Process Priors, and supplemented by additional parameters as necessary. They are:

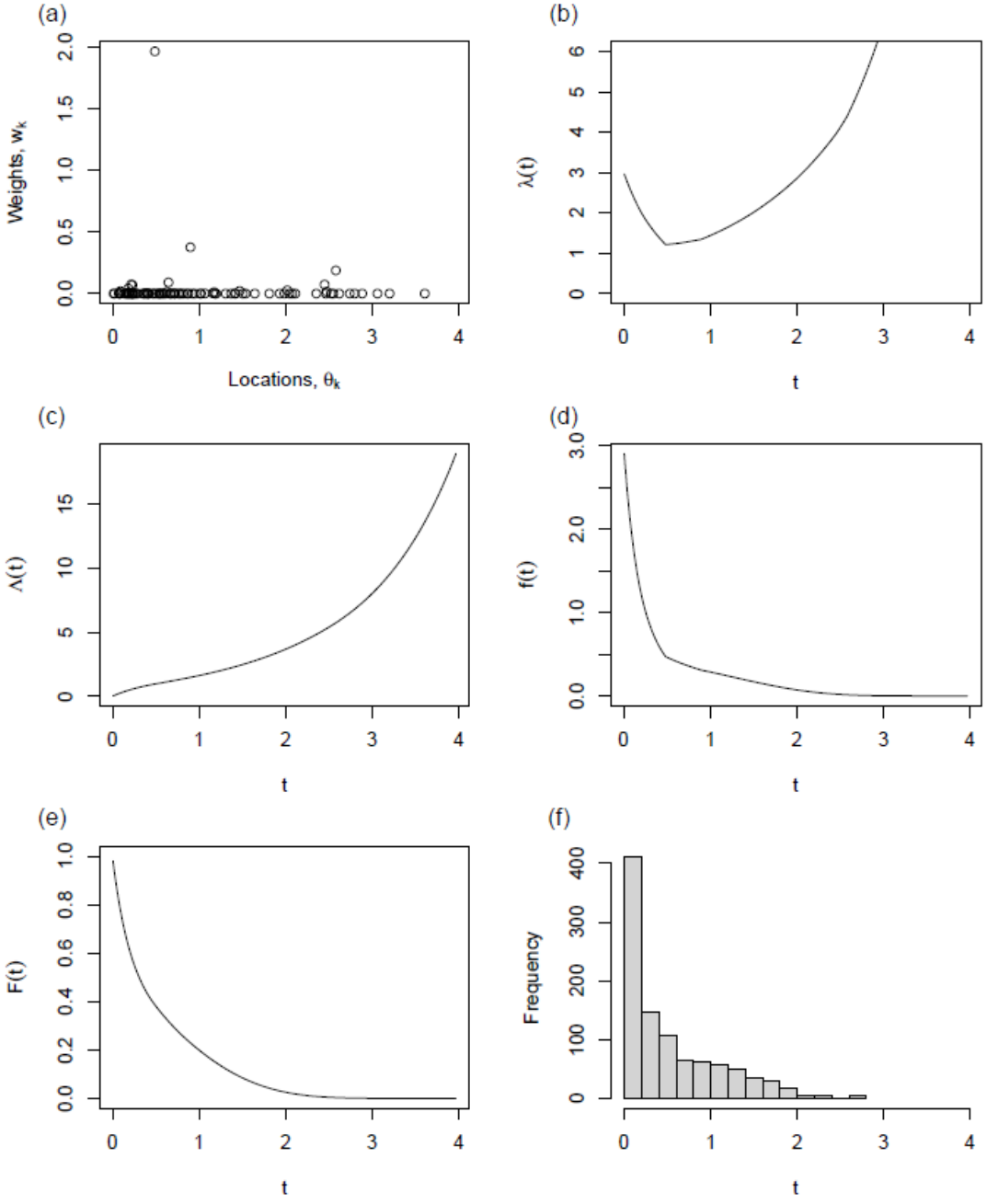
1) Increasing Failure Rate (the lower tail integral of a Gamma Process)
2) Decreasing Failure Rate (the upper tail integral of a Gamma Process)
3) Lo-Weng Bathtub (a symmetric integral of a Gamma Process)
4) Superposition Bathtub (a sum of an IFR and a DFR hazard rate)
5) Mixture Bathtub (a probabilistic mixture of an IFR and DFR distributions)
6) Log-Convex Hazard Rate (the log hazard rate has a non-decreasing gradient)

We have implemented simulation and inference for all of these models. The graphs below show example draws from the 6) Log-Convex Hazard Rate model. In this model the gradient of the log hazard rate is incremented by random positive weights at random locations (times). The graphs show the location/weight pairs, the corresponding hazard rate function, the cumulative hazard rate, the density function and the survival function, and a histogram of n=1000 random draws from the resulting failure time distribution.

Our simulation results are presented in a preprint on the arXiv (Arnold, R., Chukova, S., and Hayakawa, Y. (2023). "Bayesian non-parametric specification of bathtub shaped hazard rate functions," arXiv, pp.1-20, doi: 10.48550/arXiv.2305.08015.), and we have a paper on inference in preparation.

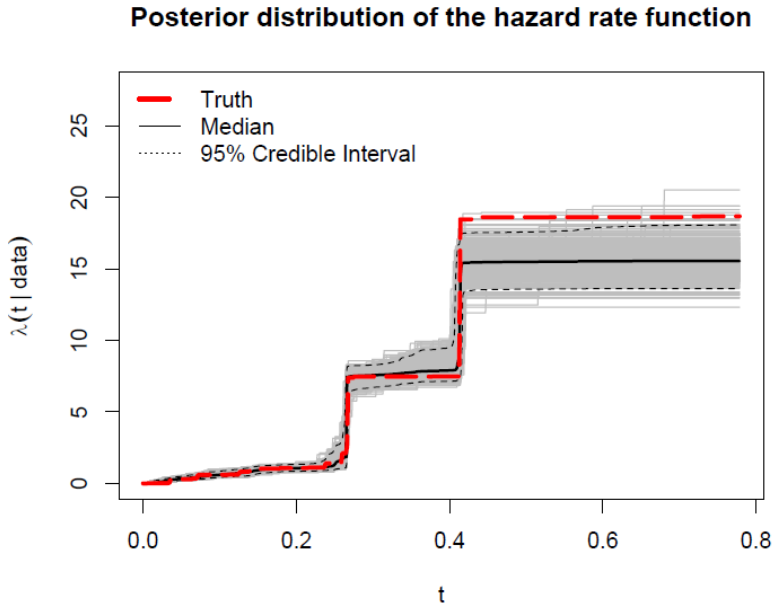
(2) Non-parametric Bayesian inference of IFR hazard rate functions using the Gamma Process Prior

We present inference for a non-parametric hazard rate function drawn from a Gamma Process Prior. We use a gamma-scaled Dirichlet Process prior to implement the Gamma Process prior, and construct a Markov Chain Monte Carlo sampler to carry out inference.



Model 6. Log Convex (LCV): A single draw from the prior, and a sample of simulated data. (a) Locations and weights $\{(\theta_k, w_k)\}$; (b) Hazard rate function $\lambda(t)$; (c) Cumulative hazard rate function $\Lambda(t)$; (d) Probability density $f(t)$; (e) Survival Function $\bar{F}(t)$; (f) Sample of $n = 1000$ failure time observations.

The following figure shows the results of implementing five parallel MCMC chains each with one million samples. This work is reported in the journal article (Arnold, R., Chukova, S. and Hayakawa, Y., "Nonparametric Bayesian Analysis of Hazard Rate Functions using the Gamma Process Prior," 2020 Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM), Vancouver, BC, Canada, 2020, pp. 1-6, doi: 10.1109/APARM49247.2020.9209405.).



Fitted hazard rate function. The thin grey lines are 500 individual draws from the posterior for the hazard rate function $\lambda(t)$. The solid and dotted lines show the median and 95% credible interval at each time point, and the long dashed line shows the true hazard rate function from which the data were actually drawn.

The following research results are from the projects which are complementary to our original goals.

(3) **Delayed reporting of faults in warranty claims**

We wrote a paper on delayed reporting of faults in warranty claims, in which the reporting process is modelled as a stochastic process dependent on the underlying stochastic process generating the faults.

(4) **Geometric-like processes**

We worked on a review paper and a book chapter on geometric-like processes and their applications in warranty analysis.

(5) **Nonzero repair times dependent on the failure hazard**

We wrote a paper on repairable systems whose repair times depend on the hazard rate.

(6) **Mean and variance of an alternating geometric process**

We introduced two new approaches for computing the mean and variance functions of two counting processes related to the alternating geometric process.

(7) **The alternating alpha-series process**

We wrote a book chapter on the alternating alpha-series process which can be used to describe the operative and repair times of a repairable system.

５．主な発表論文等

〔雑誌論文〕　計11件（うち査読付論文　8件／うち国際共著　11件／うちオープンアクセス　1件）

１．著者名 Arnold, R., Chukova, S., and Hayakawa, Y.	４．巻 -
２．論文標題 Bayesian non-parametric specification of bathtub shaped hazard rate functions	５．発行年 2023年
３．雑誌名 arXiv	６．最初と最後の頁 1-20
掲載論文のDOI（デジタルオブジェクト識別子） 10.48550/arXiv.2305.08015	査読の有無 無
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Richard Arnold, Stefanka Chukova, Yu Hayakawa and Sarah Marshall	４．巻 -
２．論文標題 Mean and variance of an alternating geometric process: An application in warranty cost analysis	５．発行年 2021年
３．雑誌名 Quality and Reliability Engineering International	６．最初と最後の頁 1-18
掲載論文のDOI（デジタルオブジェクト識別子） 10.1002/qre.2964	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 Richard Arnold, Stefanka Chukova, Yu Hayakawa and Sarah Marshall	４．巻 -
２．論文標題 Alternating Alpha-Series Process	５．発行年 2021年
３．雑誌名 Proceedings of the Reliability and Maintenance Engineering Summit 2021 (RMES 2021), 11-13 September 2021	６．最初と最後の頁 32-39
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 無
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 Richard Arnold, Stefanka Chukova and Yu Hayakawa	４．巻 -
２．論文標題 Nonparametric Bayesian Analysis of Hazard Rate Functions using the Gamma Process Prior	５．発行年 2020年
３．雑誌名 2020 Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM)	６．最初と最後の頁 1-6
掲載論文のDOI（デジタルオブジェクト識別子） 10.1109/APARM49247.2020.9209405	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Richard Arnold, Stefanka Chukova, Yu Hayakawa and Sarah Marshall	4．巻 -
2．論文標題 Mean and Variance of an Alternating Geometric Process	5．発行年 2020年
3．雑誌名 2020 Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM)	6．最初と最後の頁 1-5
掲載論文のDOI（デジタルオブジェクト識別子） 10.1109/APARM49247.2020.9209562	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Richard Arnold, Stefanka Chukova, Yu Hayakawa and Sarah Marshall	4．巻 201
2．論文標題 Geometric-Like Processes: An Overview and Some Reliability Applications	5．発行年 2020年
3．雑誌名 Reliability Engineering & System Safety	6．最初と最後の頁 -
掲載論文のDOI（デジタルオブジェクト識別子） 10.1016/j.ress.2020.106990	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Richard Arnold, Stefanka Chukova, Yu Hayakawa and Sarah Marshall	4．巻 36(3)
2．論文標題 Nonzero repair times dependent on the failure hazard	5．発行年 2020年
3．雑誌名 Quality and Reliability Engineering International	6．最初と最後の頁 988-1004
掲載論文のDOI（デジタルオブジェクト識別子） 10.1002/qre.2611	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Richard Arnold, Stefanka Chukova and Yu Hayakawa	4．巻 37
2．論文標題 Discussion of “Virtual age, is it real?”	5．発行年 2021年
3．雑誌名 Applied Stochastic Models in Business and Industry	6．最初と最後の頁 26-29
掲載論文のDOI（デジタルオブジェクト識別子） 10.1002/asmb.2585	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Richard Arnold, Stefanka Chukova and Yu Hayakawa	4．巻 -
2．論文標題 Delayed Reporting of Faults in Warranty Claims	5．発行年 2019年
3．雑誌名 IEEE Transactions on Reliability (Early Access)	6．最初と最後の頁 1〜17
掲載論文のDOI（デジタルオブジェクト識別子） 10.1109/TR.2019.2957503	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Arnold, R., Chukova, S., Hayakawa, Y. and Marshall, S.	4．巻 -
2．論文標題 Warranty cost analysis with an alternating geometric process	5．発行年 2019年
3．雑誌名 Journal of Risk and Reliability	6．最初と最後の頁 -
掲載論文のDOI（デジタルオブジェクト識別子） 10.1177/1748006X18820379	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Arnold, R., Chukova, S., Hayakawa, Y. and Marshall, S.	4．巻 1
2．論文標題 Warranty Cost Analysis with an Alternating Geometric Process	5．発行年 2018年
3．雑誌名 Proceedings of APARM2018 & QR2MSE2018	6．最初と最後の頁 327-333
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 無
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

〔学会発表〕　計19件（うち招待講演　0件／うち国際学会　17件）

1．発表者名 Sarah Marshall (Authors: Richard Arnold, Stefanka Chukova, Yu Hayakawa and Sarah Marshall)
2．発表標題 Modelling warranty claims using an alternating alpha-series process
3．学会等名 The 10th Asia-Pacific International Symposium on Advanced Reliability and Maintenance (APARM 2022)（国際学会）
4．発表年 2022年

１．発表者名
Sarah Marshall (Authors: Sarah Marshall, Richard Arnold, Stefanka Chukova, and Yu Hayakawa)

２．発表標題
Modelling warranty claims for ageing repairable systems with non-zero repair times

３．学会等名
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４．発表年
2023年

１．発表者名
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２．発表標題
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３．学会等名
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１．発表者名
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２．発表標題
Gamma Process Priors for Hazard Rate Functions

３．学会等名
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４．発表年
2021年

１．発表者名
Sarah Marshall (Authors: Richard Arnold, Stefanka Chukova, Yu Hayakawa and Sarah Marshall)

２．発表標題
Alternating Alpha-Series Process: Properties and Applications

３．学会等名
31st European Conference on Operational Research (EURO 2021)（国際学会）

４．発表年
2021年

１．発表者名
Yu Hayakawa (Authors: Richard Arnold, Stefanka Chukova, Yu Hayakawa and Sarah Marshall)

２．発表標題
Alternating Alpha-Series Process

３．学会等名
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2021年

１．発表者名
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２．発表標題
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３．学会等名
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2020年

１．発表者名
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２．発表標題
Mean and Variance of an Alternating Geometric Process

３．学会等名
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４．発表年
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１．発表者名
Richard Arnold (Authors: Richard Arnold and Yu Hayakawa)

２．発表標題
Biclustering in Capture-Recapture Experiments

３．学会等名
Sixth Symposium on Games and Decisions in Reliability and Risk（国際学会）

４．発表年
2019年

１．発表者名
Richard Arnold (Authors: Richard Arnold and Yu Hayakawa)

２．発表標題
Biclustering in Capture-Recapture Experiments

３．学会等名
New Zealand Statistical Association Conference 2019（国際学会）

４．発表年
2019年

１．発表者名
Sarah Marshall (Authors: Sarah Marshall, Richard Arnold, Stefanka Chukova and Yu Hayakawa)

２．発表標題
Computation of the Mean Value Function of an Alternating Geometric Process

３．学会等名
2019 AUT Mathematical Sciences Symposium（国際学会）

４．発表年
2019年

１．発表者名
Sarah Marshall (Authors: Sarah Marshall, Richard Arnold, Stefanka Chukova and Yu Hayakawa)

２．発表標題
Mean Value Function of an Alternating Geometric Process

３．学会等名
Operations Research Society of New Zealand Conference 2019（国際学会）

４．発表年
2019年

１．発表者名
Richard Arnold (Authors: Arnold, R. and Hayakawa, Y.)

２．発表標題
Biclustering in Capture-recapture Experiments

３．学会等名
11th International Conference on Mathematical Methods in Reliability（国際学会）

４．発表年
2019年

１．発表者名
Stefanka Chukova (Authors: Arnold, R., Chukova, S., Hayakawa, Y. and Marshall, S.)

２．発表標題
Geometric and Geometric-like Processes: An Overview and Some Applications

３．学会等名
11th International Conference on Mathematical Methods in Reliability（国際学会）

４．発表年
2019年

１．発表者名
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２．発表標題
Nonzero Repair Times Dependent on the Failure Hazard

３．学会等名
11th International Conference on Mathematical Methods in Reliability（国際学会）

４．発表年
2019年

１．発表者名
Yu Hayakawa (Authors: Arnold, R., Chukova, S. and Hayakawa, Y.)

２．発表標題
Delayed reporting of faults in warranty claims

３．学会等名
Spring Meeting of the Operations Research Society of Japan

４．発表年
2019年

１．発表者名
Hayakawa (Authors: Arnold, R., Chukova, S. and Hayakawa, Y.)

２．発表標題
Delayed reporting of faults in warranty claims

３．学会等名
Waseda International Symposium

４．発表年
2019年

１．発表者名 Sarah Marshall (Authors: Arnold, R., Chukova, S., Hayakawa, Y. and Marshall, S.)
２．発表標題 Modelling a renewing free repair warranty using an alternating geometric process
３．学会等名 2018 Joint New Zealand Statistical Association and Operations Research Society of New Zealand Conference（国際学会）
４．発表年 2018年

１．発表者名 Sarah Marshall (Authors: Arnold, R., Chukova, S., Hayakawa, Y. and Marshall, S.)
２．発表標題 Warranty Cost Analysis with an Alternating Geometric Process
３．学会等名 8th Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling（国際学会）
４．発表年 2018年

〔図書〕　計1件

１．著者名 Eds Ram, M. and Singh, S.B.; Authors of Chapter 1: Arnold, R., Chukova, S., Hayakawa, Y. and Marshall, S.	４．発行年 2019年
２．出版社 CRC Press	５．総ページ数 -
３．書名 Mathematics Applied to Engineering and Management (Chapter 1: Geometric and Geometric-Like Processes and their Applications in Warranty Analysis)	

〔産業財産権〕

〔その他〕

―

６．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考

７．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計0件

８．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関

ニュージーランド	Victoria University of Wellington	Auckland University of Technology		