様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９　(共通)

科研費 KAKENHI

科学研究費助成事業　　研究成果報告書

令和　６ 年　６ 月　３ 日現在

機関番号：１４３０１
研究種目：基盤研究(C)（一般）
研究期間：2019～2023
課題番号：１９Ｋ０３５４０

研究課題名（和文）Stochastic processes associated with resistance forms

研究課題名（英文）Stochastic processes associated with resistance forms

研究代表者
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研究者番号：５０８２４１８２

交付決定額（研究期間全体）：（直接経費）　3,300,000円

研究成果の概要（和文）：その多くは数理物理学の問題に動機づけられた，ランダム環境における様々なランダムウォークの研究を行った．重要な新しい結果が証明された重要な例の一つは，ユークリッド空間に制約されたランダムツリーの基本的な例である2次元および3次元の一様全域木である．新しい結果が得られたもう一つのモデルはモット・ランダムウォークであり，これは不均一媒質中の電子輸送のモデルである．いずれの例においても，スケール極限が得られ，その解析のテーマの一つが抵抗形式の使用であった．抵抗形式は元々フラクタル上の解析学の文脈で開発されたものであるが，現在ではランダムな環境を理解するのに有用であると考えられている．

研究成果の学術的意義や社会的意義
The motivation for studying random walks in random environments is to provide into the transport properties of disordered media. The results of this project focussed on regimes where anomalous behaviour can be observed, and thus it helps explain what features lead to a break from typical behaviour.

研究成果の概要（英文）：A range of random walks in random environments were studied, many motivated by problems in mathematical physics. One of the important examples for which significant new results were proved was the uniform spanning tree in two and three dimensions, which is a fundamental example of a random tree constrained by Euclidean space and arises as a limit of a certain model in statistical physics. Another model for which new results were obtained was Mott variable range hopping, which is a model for electron transport in inhomogeneous media. In both cases, scaling limits were obtained that show the long-time behaviour of the processes in question. Other models considered included the random conductance model, the Bouchaud trap model and percolation on certain random planar maps. In all cases, a theme of the analysis was the use of resistance forms, which were originally developed in the context of analysis on fractals, but are now being seen to be useful for understanding random environments.

研究分野：Probability theory

キーワード：random walks　random graphs　subdiffusive behaviour　uniform spanning trees　Mott random walk　random conductance model　heat kernel estimates　scaling limits

１．研究開始当初の背景（Initial background of the study）

As described in the original proposal, the focus of this project was in random walks in random environments, as motivated by wanting to describe properties of disordered media. Examples of such arise in statistical physics, such as percolation clusters and uniform spanning trees, or computer science, where various network models are applied. The particular regime of interest was for models which demonstrate some kind of large-scale fractal behavior, for which anomalous behavior of the associated random walks is expected. The study of random walks on fractal-like graphs and their scaling limits was initiated in the 1980s, and since then, more general techniques based on “resistance forms”, as introduced by Kigami, have been developed to understand them. The goal of this project was to further advance these techniques to cover more complex and challenging examples.

２．研究の目的（Objectives of the research）

The proposal contained several detailed aims, which we recall the topics of here (see the original proposal for details), and briefly describe progress made upon them.

Aim 1. Apply resistance form framework to deduce random walk scaling limits

(a) Random walk on the critical percolation IIC

In the original proposal, various problems relating to understanding random walks on critical percolation clusters conditioned to be large were indicated. Whilst the Euclidean setting still remains open, in this direction, together with Eleanor Archer (Paris Nanterre), I demonstrated the scaling limit expected to be seen in high-dimensions is seen for a related model based on random planar maps. This model could be seen as an interpolation between critical trees, as had been studied previously, and the high-dimensional Euclidean case. It suggests the general techniques for studying random walks will be robust enough to handle the Euclidean case once the geometry is better understood.

(b) Random walk on uniform spanning trees (USTs)

Together with Martin Barlow (UBC) and Takashi Kumagai (Waseda), we derived detailed heat kernel estimates for the two-dimensional UST, which show an interesting difference between the quenched (typical) and annealed (averaged) behavior. In a subsequent paper with Daisuke Shiraishi (Kyoto) and Satomi Watanabe (CUHK), we explored this phenomenon in more detail for a simpler (but strongly) related model of random walk on a loop-erased random walk. As for the case of the three-dimensional UST, together with Omer Anger (UBC), Sarai Hernandez-Torres (UPAM) and Daisuke Shiraishi (Kyoto), we adapted the two-dimensional arguments to derive a scaling limit for the associated random walk.

(c) Biased random walk on spatially embedded graphs

Together with Adam Bowditch (UCD), I studied the biased random walk on supercritical percolation, demonstrating non-Gaussian fluctuations occur in a certain part of the ballistic regime. This aspect of the problem was more challenging than previous results on non-Gaussian fluctuations in the sub-ballistic regime, since it required a detailed understanding of the second-order behavior of the model.

(d) Mott process in one dimension and in more general spaces

Together with Ryoki Fukushima (Tsukuba) and Stefan Junk (Gakushuin), I wrote two papers on the Mott process in one dimension, describing in detail the kinds of behavior that can occur in the zero-speed regime. The proofs depended on a careful analysis of the state-space as a resistance network. In a related work for a simpler one-dimensional random conductance model, with Daniel Kious (Bath) and Carlo Scali (TUM), I studied an 'aging' property, which might also be expected to be seen in the Mott model.

Aim 2. Apply resistance form framework to study properties of the limiting processes

(a) Heat kernel estimates

Over the course of the project, I worked on several papers concerning heat kernel estimates. Firstly, with Ben Hambly (Oxford) and Takashi Kumagai (Waseda), we derived heat kernel estimates for the one-dimensional FIN process. An interesting aspect of these was a 'law of large numbers'-type result for the off-diagonal part of the heat kernel, which demonstrated how the fluctuations of the environment even out over a certain time-space scale. With Hao Van Can (Vietnam Academy of Science and Technology) and Takashi Kumagai (Waseda), I derived new results on the spectral dimension of simple random walk on a long-range percolation cluster. Moreover, with Sebastian Andres (Braunschweig) and Takashi Kumagai (Waseda), I gave some quantitative local limit theorems for the one-dimensional Bouchaud trap model. In each of the three cases mentioned, although the particular models studied were quite simple, we expect the techniques to be adaptable to more complex settings.

(b) Cover times

Although on this part of the project, I was not able to complete an article before the end of the grant, together with George Andriopoulos (NYU Abu Dhabi), Laurent Menard (Paris Nanterre) and Vlad Margarint (Charlotte), I will shortly release one on the cover time of the Brownian continuum random tree (CRT), which is a canonical example of a random fractal. In particular, together with known scaling results for random walks and their local times, our result will imply convergence of cover times for various graphs that have the Brownian CRT as their scaling limit, as is expected to be the case for high-dimensional critical percolation.

(c) Trapping phenomena

Trapping phemomenon appeared in a number of the studies already outlined above, and indeed various aspects of the anomalous random behaviour seen could be attributed to it. A summary of some part of this story is provided by a survey article I wrote with Sebastian Andres (Braunschweig) and Takashi Kumagai (Waseda).

３．研究の方法 (Research methods)

As per the original plan, the main approach of this project was to develop resistance form techniques to understand the various models of interest. In general, the PI attempted to balance the abstract theoretical arguments with the study of particular models. In this way, it could be ensured that the technical conditions assumed in the general results were checkable for models of genuine interest.

４．研究成果 (Research results)

These have been outlined in part 2 of the report. Here, I would stress that, whilst most of the results derived are of interest in their own right, a number were obtained for 'simple models', and can therefore be seen as test cases for more challenging settings. The results of the project thus naturally give a number of interesting directions for future research, many of which I plan to work on further with my collaborators.

５．主な発表論文等

〔雑誌論文〕　計9件（うち査読付論文　9件／うち国際共著　7件／うちオープンアクセス　0件）

１．著者名 D. A. Croydon and D. Shiraishi	４．巻 59
２．論文標題 Scaling limit for random walk on the range of random walk in four dimensions	５．発行年 2023年
３．雑誌名 Annales de l'institut Henri Poincare (B) Probabilites et Statistiques	６．最初と最後の頁 166-184
掲載論文のDOI（デジタルオブジェクト識別子） 10.1214/22-AIHP1243	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 ―

１．著者名 M. T. Barlow, D. A. Croydon, T. Kumagai	４．巻 181
２．論文標題 doi.org/10.1007/s00440-021-01078-w	５．発行年 2021年
３．雑誌名 Probability Theory and Related Fields	６．最初と最後の頁 57,111
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s00440-021-01078-w	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

１．著者名 D. A. Croydon, D. Shiraishi	４．巻 -
２．論文標題 Scaling limit for random walk on the range of random walk in four dimensions	５．発行年 2022年
３．雑誌名 Annales de l'institut Henri Poincare (B) Probabilites et Statistiques	６．最初と最後の頁 -
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１．著者名 O. Angel, D. Croydon, S. Hernandez-Torres and D. Shiraishi	４．巻 -
２．論文標題 Scaling limits of the three-dimensional uniform spanning tree and associated random walk	５．発行年 2021年
３．雑誌名 Annals of Probability	６．最初と最後の頁 -
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 O. Angel, D. Croydon, S. Hernandez-Torres and D. Shiraishi	4．巻 -
2．論文標題 The number of spanning clusters of the uniform spanning tree in three dimensions	5．発行年 2021年
3．雑誌名 Advanced Studies in Pure Mathematics	6．最初と最後の頁 -
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 ―

1．著者名 D. Croydon and M. Holmes	4．巻 375
2．論文標題 Biased random walk on the trace of biased random walk on the trace of...	5．発行年 2020年
3．雑誌名 Communications in Mathematical Physics	6．最初と最後の頁 1341
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s00220-019-03585-3	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 David Croydon	4．巻 VI
2．論文標題 The random conductance model with heavy tails on nested fractal graphs	5．発行年 2020年
3．雑誌名 Fractal Geometry and Stochastics	6．最初と最後の頁 -
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Croydon David A.、Fukushima Ryoki、Junk Stefan	4．巻 33
2．論文標題 Anomalous scaling regime for one-dimensional Mott variable-range hopping	5．発行年 2023年
3．雑誌名 The Annals of Applied Probability	6．最初と最後の頁 4044-4090
掲載論文のDOI（デジタルオブジェクト識別子） 10.1214/22-AAP1915	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

1．著者名 Andres Sebastian、Croydon David、Kumagai Takashi	4．巻 -
2．論文標題 Heat Kernel Fluctuations for Stochastic Processes on Fractals and Random Media	5．発行年 2023年
3．雑誌名 From Classical Analysis to Analysis on Fractals	6．最初と最後の頁 265～281
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/978-3-031-37800-3_12	査読の有無 有
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 該当する

〔学会発表〕　計18件（うち招待講演　18件／うち国際学会　1件）

1．発表者名 David Croydon
2．発表標題 Random walks on random graphs in critical regimes
3．学会等名 New Horizons in Motions in Random Media（招待講演）
4．発表年 2022年～2023年

1．発表者名 David Croydon
2．発表標題 Sub-diffusive scaling regimes for one-dimensional Mott variable-range hopping
3．学会等名 University of Oxford Probability Seminar（招待講演）
4．発表年 2022年～2023年

1．発表者名 David Croydon
2．発表標題 Anomalous scaling regime for one-dimensional Mott variable-range hopping
3．学会等名 One World Probability Seminar（招待講演）
4．発表年 2022年～2023年

１．発表者名 D. A. Croydon
２．発表標題 Central limit theorem for the spectrum of a random fractal string
３．学会等名 University of Bristol, Analysis and Geometry Seminar（招待講演）
４．発表年 2021年～2022年

１．発表者名 D. A. Croydon
２．発表標題 Anomalous scaling regimes for one-dimensional random walks
３．学会等名 Isaac Newton Institute: Fractional kinetics, hydrodynamic limits and fractals（招待講演）
４．発表年 2021年～2022年

１．発表者名 David Croydon
２．発表標題 Scaling limits of the two- and three-dimensional uniform spanning trees
３．学会等名 University of Oxford, Discrete Mathematics and Probability Seminar（招待講演）
４．発表年 2020年～2021年

１．発表者名 David Croydon
２．発表標題 Anomalous scaling regime for one-dimensional Mott variable-range hopping
３．学会等名 Probability Victoria Seminar（招待講演）
４．発表年 2020年～2021年

１．発表者名
David Croydon

２．発表標題
Scaling limits of the two- and three-dimensional uniform spanning trees and the associated random walks

３．学会等名
New York University, Courant Institute, Probability and Mathematical Physics Seminar（招待講演）

４．発表年
2019年～2020年

１．発表者名
David Croydon

２．発表標題
Random walks on the two- and three-dimensional uniform spanning trees

３．学会等名
Kansai University, International workshop on stochastic analysis and applications（招待講演）

４．発表年
2019年～2020年

１．発表者名
David Croydon

２．発表標題
Scaling limits of random walks on random graphs in critical regimes

３．学会等名
Kanazawa University, Mathematical Society of Japan autumn meeting（招待講演）

４．発表年
2019年～2020年

１．発表者名
David Croydon

２．発表標題
Random walks on the two- and three-dimensional uniform spanning trees

３．学会等名
Fukuoka University, Japanese-German open conference on stochastic analysis（招待講演）

４．発表年
2019年～2020年

1．発表者名
David Croydon

2．発表標題
Random walks on fractals and critical random graphs

3．学会等名
PUC/Universidad de Chile, Probability seminar（招待講演）

4．発表年
2019年～2020年

1．発表者名
David Croydon

2．発表標題
Quenched and averaged tails of the heat kernel of the two-dimensional uniform spanning tree

3．学会等名
Northwestern University, 41st stochastic processes and their applications conference（招待講演）（国際学会）

4．発表年
2019年～2020年

1．発表者名
David Croydon

2．発表標題
Scaling limits of random walks on random graphs in critical regimes

3．学会等名
Kyoto University, Joint colloquium of the Mathematics Department/Research Institute for Mathematical Sciences（招待講演）

4．発表年
2019年～2020年

1．発表者名
David Croydon

2．発表標題
Quenched and averaged tails of the heat kernel of the two-dimensional uniform spanning tree

3．学会等名
Kobe University, Workshop on probabilistic potential theory and related fields（招待講演）

4．発表年
2019年～2020年

１．発表者名 David Croydon
２．発表標題 Random walk on a critical percolation cluster on a random hyperbolic half-planar triangulation
３．学会等名 Westlake University, China, 1st NYUSh-Peking-Westlake Joint Conference on Probability（招待講演）
４．発表年 2023年～2024年

１．発表者名 David Croydon
２．発表標題 Random walk on a critical percolation cluster on a random hyperbolic half-planar triangulation.
３．学会等名 National University of Singapore, Random Interacting Systems, Scaling Limits, and Universality: Workshop on Random Interacting Systems（招待講演）
４．発表年 2023年～2024年

１．発表者名 David Croydon
２．発表標題 Random walk on a critical percolation cluster on a random hyperbolic half-planar triangulation
３．学会等名 Kyoto University, 21st Symposium on Stochastic Analysis on Large Scale Interacting Systems（招待講演）
４．発表年 2023年～2024年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

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６．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考

７．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計0件

８．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関