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研究課題名（和文）Mathematical foundations for the reconfiguration paradigm

研究課題名（英文）Mathematical foundations for the reconfiguration paradigm

研究代表者
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研究成果の概要（和文）：ハイブリッドダイナミック論理は、再構成可能な機能を持つシステムの記述とその特性に関する推論に適していると認識されています。このプロジェクトでは、ハイブリッドダイナミック論理の統一的な研究を通じて、再構成可能なシステムの形式仕様と検証のための数学的基盤を開発しました。結果は、階層化された制度の定義によって提供される抽象的な枠組み内で開発されました。

研究成果の学術的意義や社会的意義
We are witnessing a proliferation of software-driven devices that often exhibit reconfigurable features. The operation of such devices is safety-critical, necessitating rigorous development assisted by formal methods. This project set the foundation for such formal methods.

研究成果の概要（英文）：Hybrid-dynamic logics are recognized as suitable for describing systems with reconfigurable features and reasoning about their properties. The project developed a mathematical foundation for the formal specification and verification of reconfigurable systems through a unified study of hybrid-dynamic logics. The results were developed within an abstract framework provided by the category-based definition of stratified institutions. The project centered on two main research directions:
(1) We investigated the model-theoretic properties of relatively unconventional (and therefore not well-studied) hybrid-dynamic logics, and
(2) We provided proof-theoretic results such as completeness for hybrid-dynamic logics.

研究分野：logic, category theory, formal methods

キーワード：hybrid logic　dynamic logic　institution　forcing　completeness　compactness　omitting types　interpolation

1. 研究開始当初の背景

Recent developments of computer systems have triggered a paradigm shift from standard applications with fixed resources to *reconfigurable systems*, i.e. applications that work in different operation modes, called configurations, and react flexibly to internal and external stimuli for optimizing their performance. Very often the operation of reconfigurable systems is *safety-critical*: any malfunction may result in the serious injury to people. The safety requirements can be fulfilled only by applying *formal methods*. The project provides a solid *logic-based foundation* for developing formal methods to describe reconfigurable systems and to reason formally about their properties.

Reconfigurable systems can be conceptualized as transition systems in the following manner:

(a) the configurations are states, and

(b) switching from one configuration to another is a transition.

Hybrid-dynamic logics, on the other hand, are modal logics that inherit features from both hybrid logics and dynamic logics. Their expressivity allows one to describe transition systems and express the dynamics of (re)configurations:

(a) The configurations of the software in use today may be modeled using first-order logic, higher-order logic, rewriting logic, order-sorted algebra etc; in this context, these logics are considered as base logics.

(b) The transition between configurations, i.e., the dynamics of configurations, can be specified by modalities over structured actions, which are which are built upon the base logic; The process of constructing a hybrid logic over a base logic is referred to in the literature as the hybridization [9].

To unify the study of hybrid logics and hybrid-dynamic logics, we employed a more abstract logic framework than hybridization given by the definition of stratified institution [1]. Through this approach, we were able to establish results for hybrid logics that cannot be obtained by hybridization of some base logic.

2. 研究の目的

The aim of the project was to provide a foundation for the formal specification and verification of reconfigurable systems. To achieve this objective, we proposed a uniform development of hybrid-dynamic logic knowledge at an abstract level, encompassing both its semantics and proof-theoretic aspects, independently of the details of concrete hybrid-dynamic logics:

(a) *Stratified institution model theory* amounts to advancing abstract model theory within the framework of stratified institutions. The project laid the foundation of a *specification methodology* through a unified study of the semantic properties of the multitude of hybrid-dynamic logics found in the literature.

(b) *Stratified institution proof theory* involves developing proof-theoretic infrastructure for hybrid-dynamic logics within the abstract framework of stratified institutions. This research constitutes the basis of a formal *verification methodology* for reconfigurable systems.

3. 研究の方法

The project is based on a *top-down approach* to logic: the hypothesis are kept as general as possible and introduced only by-need basis, and the unnecessary details of concrete logics are removed from proofs to reveal the clean causality between logical properties. There are at least two advantages of the abstract approach:

(a) a better understanding of the logic phenomenon that is not hindered by the largely irrelevant details of particular logical systems, but guided by structurally clean causality; and

(b) a unified study of the multitude of hybrid-dynamic logical systems used in mathematics and computer science.

This approach aligns with the universal logic trend [6], which examines logic from a relativistic, non-substantialist perspective. From a practical point of view, any technology based on the foundations developed in this project will be highly adaptable and open to future improvements.

4. 研究成果

The design of a formal method consists of several steps, depicted in the left node of Figure 1. Each step relies on certain model-theoretic or proof-theoretic properties, which should be defined at an appropriate level of abstraction within the framework of stratified institutions. The proof of these model-theoretic and proof-theoretic properties are carried out within the framework of stratified institutions, as depicted on the right side of Figure 1. Once proved, the abstract results are instantiated into concrete logical systems.

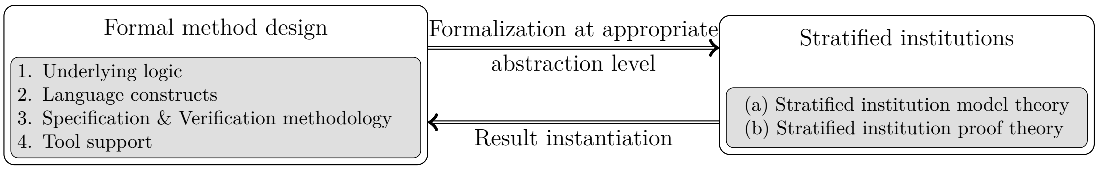

Figure 1: Formal method design

(P1) Forcing. Forcing is a method of constructing models based on consistency results. It was invented by Paul Cohen [7, 8] to prove the independence of the continuum hypothesis from the other axioms of Zermelo-Fraenkel set theory. Robinson [18] developed an analogous theory of forcing in model theory. Barwise [5] and Keisler [17] extended Robinson's theory to infinitary logic and used it to give a new proof of the Omitting Types Theorem. The present project introduces the forcing technique in the abstract framework of stratified institutions and investigates two forcing properties which are discussed below.

(P1.1) Syntactic forcing property. We study a forcing property based on a notion of syntactic consistency defined within the context of a system of proof rules defined for an abstract stratified institution. This syntactic forcing property is used to prove a layered completeness result for a logic framework given by the definition of stratified institution. The usual restriction to signatures composed of a countable number of symbols typically required for forcing is not needed in our framework. The abstract results are then instantiated into concrete examples of hybrid logics, thereby providing sound and complete entailment systems for several benchmark examples of hybrid logics. The results mentioned above are reported in [11].

(P1.2) Semantic forcing property. We develop a *forcing* technique and then we study a *forcing property* based on local satisfiability, which lead to a refined proof of the Omitting Types Theorem. Intuitively speaking, in model theory a type is a complete description in the appropriate formal language of a potential element of a model. A model may or may not have elements that satisfy such a description: if it has at least one, we say that it *realizes* the type, if it does not have any, it *omits* the type. For uncountable signatures, our result requires compactness, while for countable signatures, compactness is not necessary. We apply the Omitting Types Theorem to obtain upwards and downwards Löwenheim-Skolem theorems for several hybrid and hybrid-dynamic benchmark logics from the literature, as well as a completeness theorem for their *constructor-based* variants. The results mentioned in this paragraph were reported in [13].

(P2) Interpolation. A logical property that is useful for dealing with combining and decomposing theories is interpolation. It involves areas such as algebraic, specification, theorem proving or model checking. The reason for the interest in interpolation is the fact that it is the source of many other results. We prove the interpolation property as follows:

> Following in Lindström's footsteps, we use an Omitting Types Theorem for many-sorted hybrid-dynamic first-order logics established in [13] to obtain a Robinson Consistency Theorem. An important corollary of this result is the interpolation property. Our results rely on compactness, so they apply only to hybrid logics, since hybrid-dynamic logics are not compact.

In [2], Areces et al. solved the interpolation problem positively for hybrid propositional logic, and in [3], they establish a similar result for hybrid predicate logic with constant domains (also called here rigid domains). Our results, although similar, do not follow from theirs. For one thing, the framework of [3] is limited to rigid domains, whereas we allow variable domains. Moreover, as usual in the area of algebraic specification, we work in a many-sorted setting and, importantly, we consider arbitrary pushouts of signatures (see, e.g., [19]); not only inclusions. This seemingly small change splits one-sorted interpolation and many-sorted interpolation apart. One can have the former but not the latter. Therefore, our interpolation results are new and meaningful, and they were reported in [12].

(P3) Hybrid-Dynamic Ehrenfeucht-Fraïssé Games. In first-order logic, elementary equivalence is characterized by Ehrenfeucht-Fraïssé games [15]. While Ehrenfeucht-Fraïssé games have been applied to modal logic using the standard translation to first-order logic [10], there has been no form of Ehrenfeucht-Fraïssé game supporting different types of quantification within a modal logic context. In this project, we proposed a novel notion of modular Ehrenfeucht-Fraïssé games for hybrid-dynamic propositional logic and its fragments. Here we use Ehrenfeucht-Fraïssé games to establish a new modular Fraïssé-Hintikka Theorem for hybrid-dynamic logics and to study the relationship between countable game equivalence (determined by countable EF games), and bisimulations (determined by countable back-and-forth systems). To the best of our knowledge, there is no proof of Fraïssé-Hintikka theorem for hybrid or dynamic logics in the literature. A direct consequence of this theorem is a characterization of hybrid-dynamic elementary equivalence in terms of Ehrenfeucht-Fraïssé games.

In a Ehrenfeucht-Fraïssé game, the players' moves correspond to various types of (hybrid) quantification, including possibility over structured actions, first-order quantification, and *store*, which names the current state. The mathematical structure supporting this formalization is the gameboard tree [14], extended by labels for the edges. The role of such trees is comparable to that of a chessboard in chess. Gameboard trees can also be seen as representing the quantifier ranks of sentences, as the nodes consist of signatures, and the edges are labeled extensions of signatures with a finite set of variables. The results about hybrid-dynamic Ehrenfeucht-Fraïssé games are submitted for publication and are currently under review. The publication that made these results possible is [14].

(P4) Bisimulation. A celebrated characterization theorem due to van Benthem (see [21]) states that a first-order formula is equivalent to a translation of a modal formula if and only if it is invariant under bisimulation. Similar characterizations exist for certain fragments of hybrid logic: Areces *et al.* [2] give one for the fragment with @ and ↓, the fragment with @, was characterized by ten Cate [20], and Hodkinson and Tahiri in [16] characterized the fragment with ↓ but, importantly, without nominals. As we will see, these characterizations are rather disparate and some of them do not extend to richer fragments. In particular, the method used in [16] does not cover the language with nominals, and the authors pose the problem of finding a characterization for this fragment. They also tabulate existing results, and ask, more generally, which fragments can be characterized by some form of bisimulations. Here is a version of this table:

Hybrid signature	Invariance under
$\emptyset$	Bisimulations
$\{\downarrow\}$ w/o nominals	Quasi-injective bisimulations
$\{\downarrow\}$ with nominals	*unknown*
$\{@\}$	$\mathcal{H}(@)$-bisimulations
$\{@, \downarrow\}$	ω-bisimulations
$\{\exists\}$	*unknown*
$\{@, \exists\}$	equivalent to FOL
full signature	equivalent to FOL

In this project, we answered these questions by providing a characterization for all (sensible) fragments of the hybrid language by means of a fine-tuned version of ω-bisimulation, a notion introduced in Areces *et al.* [2], which provides one crucial ingredient for our results that have been reported in [4].

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５．主な発表論文等

〔雑誌論文〕　計8件（うち査読付論文　5件／うち国際共著　5件／うちオープンアクセス　0件）

１．著者名 Guillermo Badia, Daniel Gaina, Alex Knapp, Tomasz Kowalski, Martin Wirsing	４．巻 v1
２．論文標題 A modular bisimulation characterisation for fragments of hybrid logic	５．発行年 2024年
３．雑誌名 arXiv	６．最初と最後の頁 1~22
掲載論文のDOI（デジタルオブジェクト識別子） 10.48550/arXiv.2312.14661	査読の有無 無
オープンアクセス オープンアクセスではない、又はオープンアクセスが困難	国際共著 －

１．著者名 Guillermo Badia, Daniel Gaina, Alex Knapp, Tomasz Kowalski, Martin Wirsing	４．巻 v1
２．論文標題 Hybrid-Dynamic Ehrenfeucht-Fraisse Games	５．発行年 2023年
３．雑誌名 arXiv	６．最初と最後の頁 1~21
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 無
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１．著者名 GAINA Daniel, BADIA Guillermo, KOWALSKI Tomasz	４．巻 14
２．論文標題 Robinson consistency in many-sorted hybrid first-order logics	５．発行年 2022年
３．雑誌名 Proceedings of the Advances in Modal Logic, AiML 2022	６．最初と最後の頁 407～428
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１．著者名 GAINA Daniel、BADIA Guillermo、KOWALSKI Tomasz	４．巻 174
２．論文標題 Omitting types theorem in hybrid dynamic first-order logic with rigid symbols	５．発行年 2023年
３．雑誌名 Annals of Pure and Applied Logic	６．最初と最後の頁 103212～103212
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１．著者名 Gaina Daniel、Kowalski Tomasz	４．巻 First-view
２．論文標題 Lindstrom’s theorem, both syntax and semantics free	５．発行年 2021年
３．雑誌名 Journal of Logic and Computation	６．最初と最後の頁 1-37
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１．著者名 Gaina Daniel、Nakamura Masaki、Ogata Kazuhiro、Futatsugi Kokichi	４．巻 848
２．論文標題 Stability of termination and sufficient-completeness under pushouts via amalgamation	５．発行年 2020年
３．雑誌名 Theoretical Computer Science	６．最初と最後の頁 82～105
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１．著者名 Gaina Daniel、Kowalski Tomasz	４．巻 30
２．論文標題 Fraisse-Hintikka theorem in institutions	５．発行年 2020年
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１．著者名 Gaina Daniel	４．巻 67
２．論文標題 Forcing and Calculi for Hybrid Logics	５．発行年 2020年
３．雑誌名 Journal of the ACM	６．最初と最後の頁 1～55
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〔学会発表〕　計9件（うち招待講演　1件／うち国際学会　7件）

1．発表者名 GAINA Daniel
2．発表標題 Stability of termination under pushouts via amalgamation
3．学会等名 Australian and New Zealand Industrial and Applied Mathematics（国際学会）
4．発表年 2022年

1．発表者名 GAINA Daniel
2．発表標題 Robinson consistency in many-sorted hybrid first-order logics
3．学会等名 Advances in Modal Logic, AiML 2022（国際学会）
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1．発表者名 GAINA Daniel
2．発表標題 Horn clauses in Hybrid-Dynamic Quantum Logic
3．学会等名 Cracow Logic Conference (CLoCk)（招待講演）（国際学会）
4．発表年 2023年

1．発表者名 Daniel Gaina
2．発表標題 Omitting Types Theorem in hybrid-dynamic first-order logic with rigid symbols
3．学会等名 The Australasian Association for Logic Conference（国際学会）
4．発表年 2021年

１．発表者名
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２．発表標題
Stability of termination under pushouts via amalgamation

３．学会等名
31st Seminar on Algebra, Logic and Geometry in Informatics (ALGI 2021)

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１．発表者名
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Omitting Types Theorem in hybrid-dynamic first-order logic with rigid symbols

３．学会等名
Mathematical Society of Japan, Autumn Meeting 2021

４．発表年
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１．発表者名
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Stability of termination under pushouts via amalgamation

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１．発表者名
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1．発表者名 Daniel Gaina
2．発表標題 Forcing and its applications in hybrid-dynamic logics
3．学会等名 Australasian Logic Group（国際学会）
4．発表年 2020年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

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6．研究組織

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8．本研究に関連して実施した国際共同研究の実施状況

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