様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９　（共通）

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研究課題名（和文）Efficient Numerical Solution for Constrained Tensor Ring Decomposition: A Theoretical Convergence Analysis and Applications

研究課題名（英文）Efficient Numerical Solution for Constrained Tensor Ring Decomposition: A Theoretical Convergence Analysis and Applications

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研究成果の概要（和文）：(1) テンソルベースの最適化問題の最小二乗部分問題を解くために、ランダム化された貪欲なKaczmarz型前処理フレキシブルGMRES法を提案する。
(2) 不良設定テンソル ネットワークを扱うために、非負性を持つグラフ正則化テンソル環を考慮する。提案されたモデルはテンソル リング分解を拡張し、非負の多元データに対する強力な表現学習ツールとして機能します。

研究成果の学術的意義や社会的意義
Extracting meaningful and interpretable low-dimensional representation from high-dimensional data is a fundamental task in the fields of signal processing and machine learning. Our research provide robust mathematical tools for handling some computational and application problems.

研究成果の概要（英文）：(1) For solving the least squares subproblems of tensor-based optimization problem, we propose a randomized and greedy Kaczmarz-type inner-iteration preconditioned flexible GMRES method.
(2) For handling the ill-posed tensor network, we consider graph-regularized tensor-ring with nonnegativity. The proposed models extend tensor ring decomposition and can be served as powerful representation learning tools for non-negative multiway data.

研究分野： Numerical linear algebra

キーワード： iterative method　tensor　convergence　nonnegative　random

１．研究開始当初の背景

Research background

Multi-dimensional array or tensor such as color images, video, gene data and so on have natural constraints, such as nonnegativity, smoothness and sparsity, so it is a challenging task to perform dimensionality reduction analysis and extract low-dimensional features on these tensor data (Cichocki et al. 2015). The typical way is to matricize and vectorize the tensor data, and then a low-rank approximation method is implemented to extract low-dimensional features such as singular value decomposition (SVD), principal component analysis (PCA), and nonnegative matrix factorization (NMF), etc. However, matricization and vectorization usually destroy the structural information of data. The constrained tensor decomposition has been developed to overcome these shortcomings to obtain the better low-dimensional representation of data. For example, the existing nonnegative tensor decompositions include: nonnegative CANDECOMP/PARAFAC decomposition (NCP); nonnegative Tucker decomposition (NTD); nonnegative tensor train decomposition (NTT). Although these models have been widely used in feature extraction (Wang et al. 2011), it has limited data representation capability, and suffers from the curse of dimensionality since the number of entries of the tensor cores will increase exponentially as the tensor order increases.

In addition, the tensor-based optimization problems with Tikhonov regularization and constraints are generally ill-posed inverse problems, which require the efficient and robust iterative solvers to obtain local minima and critical points. Also, as the tensor-based optimizations are nonconvex, the sufficient conditions and assumptions are required for guaranteeing the convergence. The theoretical convergence analysis and the computational complexity are still under discussion and analysis. **Therefore, the construction and theoretical analysis of the tensor related iterative methods attract growing attentions during the recent years in multidimensional data/signal processing and machine learning scientific engineering areas.**

２．研究の目的

Research goal

The purpose of this research is to investigate efficient numerical algorithms for recovering the multilinear latent feature, alleviating the curse-of-dimensionality and providing low-rank interpretable analysis from high-dimensional data in machine learning, quantum physics, etc. Specifically, we study constrained tensor ring decomposition with theoretical analysis and numerical performance.

３．研究の方法

Research method

This research project aims to propose efficient numerical solution for constrained tensor ring decomposition, including theoretical convergence analysis on the iterative methods and their applications to high dimensional data/signal analysis. The research methods are listed as follows.

(1) For solving the least squares subproblems of tensor-based optimization problem, **we propose a randomized and greedy Kaczmarz-type inner-iteration preconditioned flexible GMRES method**.
(2) For handling the ill-posed and overfitting issues in tensor network, we consider **graph regularized tensor ring decomposition with nonnegative constraints**.
(3) As a further research work of item (2), we **continue to work on other regularization terms, e.g., total variation, of the tensor based ill-posed problem**.
(4) As a further research work of item (1), we consider to explore the efficient iterative solvers for the consistent and inconsistent linear system, including the tensor based optimization problem. We propose a class of randomized block reflective Kaczmarz method for linear systems with automatic restart.

４．研究成果

<u>Research results</u>

(1) The subproblems of constrained tensor train/ring decomposition are ill-posed inverse problem that is generally difficult to solve. In this work, we propose a parameter tuning strategy for adjusting the number of inner iterations and the relaxation parameter. The theoretical analysis of the right-preconditioned flexible GMRES is discussed. Numerical experiments on unconstrained least squares problem show that the proposed method Is superior to the classical GMRES method preconditioned by NE-SOR inner iterations in terms of total CPU time.

The work had been published on **SIAM Journal on Scientific Computing [43 (5), S345-S366]**, which is a top journal in applied and computational mathematics.

(2) Tensor-ring (TR) decomposition is a powerful tool for exploiting the low-rank property of multiway data and has been demonstrated great potential in a variety of important applications. We propose non-negative TR (NTR) decomposition and graph-regularized NTR (GNTR) decomposition are proposed. The former equips TR decomposition with the ability to learn the parts-based representation by imposing non-negativity on the core tensors, and the latter additionally introduces a graph regularization to the NTR model to capture manifold geometry information from tensor data. Both of the proposed models extend TR decomposition and can be served as powerful representation learning tools for non-negative multiway data. The optimization algorithms based on an accelerated proximal gradient are derived for NTR and GNTR. We also empirically justified that the proposed methods can provide more interpretable and physically meaningful representations. For example, they are able to extract parts-based components with meaningful color and line patterns from objects. Extensive experimental results demonstrated that the proposed methods have better performance than state-of-the-art tensor-based methods in clustering and classification tasks.

The work had been published on **IEEE Transactions on Cybernetics, [Volume: 53, Issue: 5, May 2023, DOI: 10.1109/TCYB.2022.3157133]**.

(3) The color images can be naturally represented as tensor data formats. The linear discrete ill-posed problem with total variation Tikhonov model is considered for preserving sharp attributes in images. Meanwhile, the restored images from TV-based methods are constrained in a given dynamic range. We propose using the alternating direction method of multipliers to solve the constrained models and the

constrained subproblems are solved by modulus-based iteration methods. Our numerical results show that for some images where there are many pixels with values lying on the boundary of the dynamic range, and the proposed algorithm is better than state-of-the-art unconstrained algorithms in terms of both accuracy and robustness with respect to the regularization parameter.

The work has been submitted to the **Journal of Computational and Applied Mathematics**, which is currently under review.

(4) We propose novel deterministic and randomized block reflective Kaczmarz methods, which are based on the row partitions block Householder transformation, for solving linear systems. The convergence analysis on both consistent and inconsistent cases are discussed, which sheds light on why the reflective type method may converge slower than the projection type method. Subsequently, the proposed methods are restarted automatically to further speed up the convergence. Numerical experiments on the synthetic random problems and image restoration problems show the efficiency of the proposed methods.

The work has been submitted to the **SIAM Journal on Scientific Computing**, which is currently under review.

(5) Other research activities

For other academic contributions or activities, I served as a **guest editor for the special issue "Recent advances in the numerical solution of matrix problems" (https://dl.acm.org/doi/abs/10.1016/j.cam.2023.115239) in the Journal of Computational and Applied Mathematics**. The issue collects several papers on the recent development on numerical linear algebra and numerical optimization problems.

I also serve as a member of editorial board on "Electronic Transactions on Numerical Analysis (ETNA)", which is an international journal on numerical analysis. (http://etna.mcs.kent.edu/editors/)

In summary, extracting meaningful and interpretable low-dimensional representation from high-dimensional data is a fundamental task in the fields of data mining, signal processing, and machine learning. Under the KAKENHI research program "Efficient Numerical Solution for Constrained Tensor Ring Decomposition: A Theoretical Convergence Analysis and Applications", we fulfill the aims and tasks by proposing efficient iterative solvers and preconditioners for constrained tensor-based ill-posed problems on both theoretical analysis and practical applications. Apart from the submitted and under review work, we will further study the rank adaption problem in tensor network and develop efficient alternating direction method of multipliers (ADMM) related algorithms for tensor data/signal processing. Meanwhile, the consideration of real application problem including the high dimensional tensor point cloud and computer vision will also benefit to our theoretical analysis.

５．主な発表論文等

〔雑誌論文〕　計5件（うち査読付論文　5件／うち国際共著　5件／うちオープンアクセス　5件）

１．著者名 Yu Yuyuan、Zhou Guoxu、Zheng Ning、Qiu Yuning、Xie Shengli、Zhao Qibin	４．巻 2
２．論文標題 Graph-Regularized Non-Negative Tensor-Ring Decomposition for Multiway Representation Learning	５．発行年 2022年
３．雑誌名 IEEE Transactions on Cybernetics	６．最初と最後の頁 1～14
掲載論文のDOI（デジタルオブジェクト識別子） 10.1109/TCYB.2022.3157133	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Sugihara Kota、Hayami Ken、Zheng Ning	４．巻 27
２．論文標題 Right preconditioned MINRES for singular systems　　†	５．発行年 2020年
３．雑誌名 Numerical Linear Algebra with Applications	６．最初と最後の頁 e2277
掲載論文のDOI（デジタルオブジェクト識別子） 10.1002/nla.2277	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Li Binghua、Li Chao、Duan Feng、Zheng Ning、Zhao Qibin	４．巻 -
２．論文標題 TPFN: Applying Outer Product Along Time to Multimodal Sentiment Analysis Fusion on Incomplete Data	５．発行年 2020年
３．雑誌名 European Conference on Computer Vision ECCV 2020	６．最初と最後の頁 431～447
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/978-3-030-58586-0_26	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Song Xiongfeng、Xu Wei、Hayami Ken、Zheng Ning	４．巻 85
２．論文標題 Secant variable projection method for solving nonnegative separable least squares problems	５．発行年 2020年
３．雑誌名 Numerical Algorithms	６．最初と最後の頁 737～761
掲載論文のDOI（デジタルオブジェクト識別子） 10.1007/s11075-019-00835-2	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

1．著者名 Yi-Shu Du, Ken Hayami, Ning Zheng, Keiichi Morikuni, Jun-Feng Yin	4．巻 -
2．論文標題 Kaczmarz-type inner-iteration preconditioned flexible GMRES methods for consistent linear systems	5．発行年 2021年
3．雑誌名 SIAM Journal on Scientific Computing	6．最初と最後の頁 -
掲載論文のDOI（デジタルオブジェクト識別子） なし	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

〔学会発表〕　計0件

〔図書〕　計0件

〔産業財産権〕

〔その他〕

researchmap
https://researchmap.jp/nzheng

6．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考

7．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計0件

8．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関