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研究課題名(和文) Generalizations of the double shuffle relations for multiple zeta values and the connections to modular forms

研究課題名(英文) Generalizations of the double shuffle relations for multiple zeta values and the connections to modular forms

研究代表者

BACHMANN Henrik (Bachmann, Henrik)

名古屋大学・多元数理科学研究科(国際)・G30特任准教授

研究者番号：20813372

交付決定額(研究期間全体)：(直接経費) 1,700,000円

研究成果の概要(和文)：この研究プロジェクトでは、多種ゼータ値の古典的な二重シャッフル関係の一般化が導入され、研究されました。これ新しい一連の方程式は、Gangl-Kaneko-Zagier によって導入された複数の Eisenstein 級数によって動機付けられています。という概念を使って定義されます。bimoulds であり、それらは対称でスワップ不変の bimoulds によって与えられます。

研究成果の学術的意義や社会的意義

This research project proposed a new family of relations which seem to describe exactly the relations satisfied by multiple Eisenstein series. They can be seen as a "modular" analogue of the classical double shuffle relations for multiple zeta values.

研究成果の概要(英文)：In this research project a generalization of the classical double shuffle relations of multiple zeta values were introduced and studied. This new set of equations are motivated by multiple Eisenstein series introduced by Gangl-Kaneko-Zagier. It is defined by using the notion of bimoulds and they are given by those bimoulds which are symmetric and swap invariant.

研究分野：Number theory

キーワード：Multiple zeta values Modular forms

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様式 C - 19、F - 19 - 1、Z - 19 (共通)

## 1 . 研究開始当初の背景

In the project "Generalizations of the double shuffle relations for multiple zeta values and the connections to modular forms" the connection of modular forms and multiple zeta values were studied. This research lies within the intersection of multiple zeta values their q-analogues, and modular forms. Multiple zeta values are numbers that are natural generalizations of special values of the Riemann zeta function and they appear in several different contexts, such as modular forms, knot theory, enumerative geometry, mixed Tate motives, quantum groups, moduli spaces of vector bundles, scattering amplitudes and many more. In recent years several connection of multiple zeta values to modular forms via q-analogues were observed and conjectured. The goal of this research project was to investigate some of them.

## 2 . 研究の目的

The purpose of the research project is the investigation of the relations satisfied by multiple Eisenstein series. The question was to find an analogue of the double shuffle relations in the context of modular forms.

### 3 . 研究の方法

The research was conducted by interchanging ideas with several experts in the field. Mostly by giving talks at conferences and by meeting them in person. Often the computer was used (e.g. Pari/Sage/Mathematica) to obtain conjectures of the objects.

### 4 . 研究成果

In this research project a generalization of the classical double shuffle relations of multiple zeta values were introduced and studied. This new set of equations are motivated by multiple Eisenstein series introduced by Gangl-Kaneko-Zagier. It is defined by using the notion of bimoulds and they are given by those bimoulds which are symmetril and swap invariant. Coefficients of bimoulds satisfying this property are called formal multiple Eisenstein series. We showed (j.w. A. Burmester) that the space of formal multiple Eisenstein series has a realization given by an algebra homomorphism into the space of formal  $q$ -series with rational coefficients. In depth one these are exactly Eisenstein series and their derivatives. This gives a natural bridge between the theory of modular forms and multiple zeta values. In another project (j.w. J.W. van Ittersum) we show

that the space of formal multiple Eisenstein series is an  $sl_2$ -algebra. This generalized the  $sl_2$ -modular structure of quasimodular forms and gives new insights into the study of multiple Eisenstein series and  $q$ -analogues of multiple zeta values. Another result of this study is a new notion of formal (quasi)modular forms equipped with a formal version of the natural derivation  $q d/dq$ .

5. 主な発表論文等

〔雑誌論文〕 計0件

〔学会発表〕 計4件（うち招待講演 0件 / うち国際学会 0件）

1. 発表者名 Henrik Bachmann
2. 発表標題 Combinatorial multiple Eisenstein series
3. 学会等名 Kyushu University Lecture series
4. 発表年 2022年

1. 発表者名 Henrik Bachmann
2. 発表標題 関西多重ゼータ研究会第54回
3. 学会等名 Formal multiple Eisenstein series and formal multiple zeta values
4. 発表年 2021年

1. 発表者名 Henrik Bachmann
2. 発表標題 Formal multiple Eisenstein serie
3. 学会等名 Seminar arithmetische Geometrie und Zahlentheorie, Hamburg
4. 発表年 2021年

1. 発表者名 Henrik Bachmann
2. 発表標題 The zoo of multiple Eisenstein series
3. 学会等名 多重ゼータ値の諸相
4. 発表年 2022年

〔図書〕 計0件

〔産業財産権〕

〔その他〕

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6. 研究組織

	氏名 (ローマ字氏名) (研究者番号)	所属研究機関・部局・職 (機関番号)	備考
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7. 科研費を使用して開催した国際研究集会

〔国際研究集会〕 計0件

8. 本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関
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