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研究課題名（和文）Exploring the effect of correlations on quantum speed limits in interacting cold atom systems

研究課題名（英文）Exploring the effect of correlations on quantum speed limits in interacting cold atom systems

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交付決定額（研究期間全体）：（直接経費）　　2,000,000円

研究成果の概要（和文）：研究助成金の目的は、複雑な量子系における非平衡ダイナミクスを、量子速度限界と量子制御に焦点を当てて探求することであった。これらのテーマについて、合計8本の論文が発表された。このうち2本は権威あるPhysical Review letters誌に掲載された。要約すると、このプロジェクトは、量子制御と速度限界の概念が、複雑な相互作用量子系にどのように拡張できるかを示したものであり、量子熱機関や量子電池のような、量子効果によって強化される次世代の量子デバイスを構築するために重要なものである。

研究成果の学術的意義や社会的意義
My work extending control concepts to many-particle quantum systems has many applications in future quantum devices. For instance, due to the presence of correlations these systems can show an improvement over classical ones. These techniques are readily applicable to current cold atom experiments.

研究成果の概要（英文）：The aim of the research grant was to explore nonequilibrium dynamics in complex quantum systems, with a focus on quantum speed limits and quantum control. In Total 8 papers were published on these topics, 2 in the prestigious Physical Review letters, 3 in Physical Review Research, 1 in the journal Quantum and 1 review article in Physics Reports. Finally an article was published for a special issue in New Journal of Physics on "Focus on Quantum Speed Limits and its Applications". This article is the highlight of this project, showing how quantum speed limits emerge in supersymmetric quantum mechanics and how these relate to high fidelity quantum control of these systems, and is the first work on this topic. In summary the project showed how concepts from quantum control and speed limits can be extended to complex interacting quantum systems, an important for building the next era of quantum devices such as quantum heat engines and batteries, which are enhanced due to quantum effects.

研究分野：Quantum control

キーワード：quantum control　quantum speed limits　few-body systems　many-body systems　quantum heat engine　thermodynamics　cold atoms

１．研究開始当初の背景

A fundamental concept in the control of quantum systems is how fast can they can be driven while still ensuring high fidelity. This has important implications for quantum computing where high gate fidelity is needed for accurate calculations; and quantum thermal machines, where non-equilibrium excitations can seriously impede the efficiency of heat engines and refrigerators. While finding this minimal operation time can be difficult for large many-body states with a large number of microscopic variables, qualitative insights can be gained from quantum speed limits (QSLs). Mandelstam and Tamm (MT) first used the energy variance of a quantum state to derive the QSL time, which bounds the minimal time for a quantum system to evolve between two distinct states [Mandelstam & Tamm, J. Phys. 9, 249 (1945)]. The concept of QSLs is therefore intimately linked with the thermodynamics of quantum systems, meaning that an understanding of non-equilibrium excitations and the work probability distribution is essential for designing efficient and robust quantum control methods. Significant advancements in the past few years has then led to QSLs being generalized for open systems and controlled dynamics such as shortcuts to adiabaticity (STAs), which can realize adiabatic dynamics on short times with perfect fidelity [Deffner & Campbell, J. Phys. A: Math. Theor. 50, 453001 (2017)], while recent works have also explored the role of coherence in dynamics [Marvian, Spekkens & Zanardi, Phys. Rev. A 93, 052331 (2016)] and its role in quantum control [Xu et al, Phys. Rev. Res. 2, 023125 (2020)].

２．研究の目的

While many advances have been made in relations to QSLs and quantum control, there are still open questions related to role that many-body correlations and coherences can have on the speed of evolution. Cold atom systems offer an ideal testbed for many-body QSLs as current experiments with atoms trapped in optical potentials are precise and controllable, while interparticle interactions can be tuned via Feshbach resonances which can in turn induce quantum correlations. Although in general solving these models requires large computational resources, if the system is confined to one-dimension of motion and only a few particles are considered the problem can become numerically tractable using exact diagonalization. The prupose of my project was to explore the effect of finite interactions on the dynamics of cold atom systems and how their inherent correlations can affect the speed of evolution. The dynamics that was explored was both uncontrolled, i.e. sudden or linear quenches of the systems Hamiltonian, along with controlled dynamics through the use of shortcuts to adiabaticity. The latter is important for the operation of quantum thermal machines, such as heat engines, allowing to improve efficiency and power, and will be a use case for my study.

３．研究の方法

The main works in this project covered the dynamics of isolated systems through the dynamical control of the external trapping potential or the two-body interaction term. I used exact diagonalization to numerically find the groundstate of the interacting systems, both in first quantization using the finite difference method (good for single particle states and small interacting systems), and in second quantization (effective for larger systems of 10s of interacting particles) by writing the Hamiltonian in the Fock basis. This was carried out using the high performance computing resources at OIST. For the case of quench dynamics the exact time-evolved state can be simply computed by projecting into the eigenbasis of the final Hamiltonian allowing for direct access to the work distribution function and Loschmidt echo. For time-dependent Hamiltonians, including STAs, the implicit Crank-Nicolson method was mostly used.

STAs were derived using a variety of methods. The first is a variational method, which is useful for complex systems without analytic solutions, and therefore depends on an accurate ansatz for the time evolved state. The equations of motion for the system can then be derived by minimizing the grand Lagrangian, and then inverse engineering the STA ramp. As it is approximate the STA can fail at very short driving times, but it is still better than non-optimized ramps. The second method is counter-diabatic (CD) driving, in which a term is derived which counteracts non-adiabatic excitations thereby keeping the system in the instantaneous groundstate of the driving Hamiltonian throughout the dynamics. This method is exact and therefore works for any driving time, however, the counter-diabatic term is in general non-local so has limited experimental applications.

QSLs for both quench dynamics and STAs are well known and are usually based on the Bures angle (related to the overlap of two states) and the energy variance during the dynamics. Specifically for counter-diabatic driving the QSL time depends on the energetic cost of the STA, namely the excess energy of the state induced by the counter-diabatic term. The cost can be analytically computed with knowledge of both the instantaneous groundstate of the driven Hamiltonian and the algebraic form of the CD term.

４．研究成果

I will summarize the results from the publications for this KAKENHI into two general categories of the dynamics, sudden quenches and STA.

Firstly, I will discuss the works focusing on sudden quenches, which entail the calculation of the full work distribution and how this describes the non-equilibrium dynamics. Important quantities when calculating the QSL time of quantum systems (namely MT bound) are of course the distance between two states, and the variance of the work distribution. The former vanishes when these two states, the initial state and either the target state or post quench nonequilibrium state, are orthogonal, while the latter is strongly dependent on the energy spectrum of the final Hamiltonian. In my works I have focused on complex systems with non-trivial spectra and trying to find universal features of the dynamics. In [Fogarty et al, Quantum 5, 486 (2020)] I investigated the emergence of quantum chaos in few-body interacting systems, finding the minimum number of particle needed to elicit chaotic dynamics. This work relied on analyzing the energy spectrum, showing that an abundance of avoided crossing due to the competition between interactions and the scattering from periodic potential barriers can lead to a characteristic Wigner-Dyson distribution. Analysis of the spectral form factor, or survival probability, can show signatures of chaoticity in the nonequilibrium dynamics along with reduced relaxation times. In [Mikkelsen et al, Physical Review Letters, 128, 070605 (2022)] this idea was extended to look at information scrambling and its connection to the work statistics, showing that the variance of the work distribution is directly proportional to the time-averaged squared commutator in harmonically trapped interacting systems. These works showed how finite interactions between cold atoms can result in complex, non-trivial dynamics, due to interaction induced shifts in the energy spectrum. This then has a significant impact on the QSL time, as characterized through the fidelity decay of the state, highlighting how interactions may be tuned to increase the evolution speed of cold atom systems.

In a similar manner, complex energy spectra can be created through the use of pseudo-random or quasi-periodic single particle potentials. In [Kiely et al, Physical Review Research 5, L022010 (2023)] we showed how the entropy of the work distribution following a quantum quench can signal quantum phase transitions, focussing on the Andre-Aubry model and the localization transition. Interestingly, the localization transition is not captured by the moments of the work distribution, and therefore cannot be inferred from the QSL time alone, however, it can be revealed through examination of the full work distribution. In this case the spectrum is continuous in the delocalized phase, while it is fractal in the localized phase, with quenches from one to the other saturating the entropy of the work distribution. This work highlighted how simple QSLs based on moments of the work distribution cannot capture quantum critical effects, however, more accurate bounds based on entropic quantities are more suitable.

Finally, I highlight two works based on controlled dynamics, where cold atom systems are subjected to time dependent Hamiltonians. The first, [Boubakour et al, Physical Review Research 5, 013088 (2023)] focused on designing efficient quantum heat engines whereby both the interaction and trapping potential for a few-body system are dynamically changed during the adiabatic strokes. Here, we show that while the interaction energy is small, it can have a large effect on engine performance, as it modifies the energy spectrum in a non-trivial manner. This gain can lead to more power output when compared to the dynamics of engines where the interactions are fixed, allowing to reach higher power on shorter timescales. Here, the increased speed of evolution is related to the interaction effects, as discussed earlier for quenched systems, show how a cooperative many-particle gain can be achieved in non-equilibrium processes.

Lastly, I will discuss one of the main highlights of this project, [Campbell et al, New Journal of Physics 24, 095001 (2022)], on quantum control and quantum speed limits in supersymmetric potentials. This work is a culmination of the work that came before, combining complex spectral analysis with non-equilibrium dynamics and control through STAs. Here we showed how new quantum control schemes can be designed by taking advantage of supersymmetric (SUSY) potentials. Each potential in the SUSY hierarchy is connected by unique SUSY operators, that transform states from one potential to another, crucially, at the same energy. The spectra are therefore equivalent for each SUSY potential, allowing to design an STA for one potential and then transforming it to work for any other potential that is part of the SUSY hierarchy. The QSLs of each potential are similarly related, showing how the QSL time changes depending on the quantum state, i.e. the distance, and not the energy eigenvalues, i.e. the speed. We showed that the leading term of the QSL time is constant for all SUSY potentials, only depending on the driving Hamiltonian, while the subsequent term is state and SUSY potential dependent. This work, and the unique SUSY hierarchy allow to study QSLs in a new setting, removing differences in the spectrum and allowing to focus solely on the differences in the geometric distances between states. It also shows the power of SUSY quantum mechanics in designing control protocols and has exciting applications in many-body physics, allowing to simplify control pulse design for dynamical processes.

５．主な発表論文等

〔雑誌論文〕　計8件（うち査読付論文　8件／うち国際共著　8件／うちオープンアクセス　8件）

１．著者名 Campbell Christopher、Fogarty Thomas、Busch Thomas	４．巻 4
２．論文標題 Nonequilibrium many-body dynamics in supersymmetric quenching	５．発行年 2022年
３．雑誌名 Physical Review Research	６．最初と最後の頁 33014
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevResearch.4.033014	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Campbell C、Li J、Busch Th、Fogarty T	４．巻 24
２．論文標題 Quantum control and quantum speed limits in supersymmetric potentials	５．発行年 2022年
３．雑誌名 New Journal of Physics	６．最初と最後の頁 095001～095001
掲載論文のDOI（デジタルオブジェクト識別子） 10.1088/1367-2630/ac89a4	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Boubakour Mohamed、Fogarty Thomas、Busch Thomas	４．巻 5
２．論文標題 Interaction-enhanced quantum heat engine	５．発行年 2023年
３．雑誌名 Physical Review Research	６．最初と最後の頁 13088
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevResearch.5.013088	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Kiely Anthony、O'Connor Eoin、Fogarty Thomas、Landi Gabriel T.、Campbell Steve	４．巻 5
２．論文標題 Entropy of the quantum work distribution	５．発行年 2023年
３．雑誌名 Physical Review Research	６．最初と最後の頁 L022010
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevResearch.5.L022010	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

１．著者名 Mikkelsen M, Fogarty T, Busch Th.	４．巻 128
２．論文標題 Connecting Scrambling and Work Statistics for Short-Range Interactions in the Harmonic Oscillator	５．発行年 2022年
３．雑誌名 Physical Review Letters	６．最初と最後の頁 70605
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevLett.128.070605	査読の有無 有
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１．著者名 Keller Tim, Fogarty Thomas, Busch Thomas	４．巻 128
２．論文標題 Self-Pinning Transition of a Tonks-Girardeau Gas in a Bose-Einstein Condensate	５．発行年 2022年
３．雑誌名 Physical Review Letters	６．最初と最後の頁 53401
掲載論文のDOI（デジタルオブジェクト識別子） 10.1103/PhysRevLett.128.053401	査読の有無 有
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１．著者名 Fogarty Thomas, Garcia-March Miguel Angel, Santos Lea F., Harshman Nathan L.	４．巻 5
２．論文標題 Probing the edge between integrability and quantum chaos in interacting few-atom systems	５．発行年 2021年
３．雑誌名 Quantum	６．最初と最後の頁 486-486
掲載論文のDOI（デジタルオブジェクト識別子） 10.22331/q-2021-06-29-486	査読の有無 有
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１．著者名 Mistakidis S.I.、Volosniev A.G.、Barfknecht R.E.、Fogarty T.、Busch Th.、Foerster A.、Schmelcher P.、Zinner N.T.	４．巻 1042
２．論文標題 Few-body Bose gases in low dimensions - A laboratory for quantum dynamics	５．発行年 2023年
３．雑誌名 Physics Reports	６．最初と最後の頁 1～108
掲載論文のDOI（デジタルオブジェクト識別子） 10.1016/j.physrep.2023.10.004	査読の有無 有
オープンアクセス オープンアクセスとしている（また、その予定である）	国際共著 該当する

〔学会発表〕　計3件（うち招待講演　0件／うち国際学会　0件）

1．発表者名
Thomas Fogarty

2．発表標題
Self-Pinning Transition of a Tonks-Girardeau Gas in a Bose-Einstein Condensate

3．学会等名
Ultracold atoms Japan, Okinawa, Japan

4．発表年
2022年

1．発表者名
Thomas Fogarty

2．発表標題
Cooperative effects in many-body heat engines

3．学会等名
2022 Tsung-Dao Lee Institute Youth Forum for Quantum Physics

4．発表年
2022年

1．発表者名
Thomas Fogarty

2．発表標題
The self-pinning transition and its man-body soliton dynamics

3．学会等名
QUACS: Quantum Adiabatic Control and Shortcuts

4．発表年
2023年

〔図書〕　計0件

〔産業財産権〕

〔その他〕

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6．研究組織

	氏名 （ローマ字氏名） （研究者番号）	所属研究機関・部局・職 （機関番号）	備考

7．科研費を使用して開催した国際研究集会

〔国際研究集会〕　計0件

８．本研究に関連して実施した国際共同研究の実施状況

共同研究相手国	相手方研究機関