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研究課題名（和文）離散付置環上のモチビックコホモロジー

研究課題名（英文）Motivic cohomology over discrete valuation rings

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研究成果の概要（和文）：数論幾何学とは，多項式の系の整数解または有理数解の研究である．そのために，他の領域における解の研究にも役に立つ，例えば，複素数解，実数解，有限体解，とp-進解は興味深い．この解の集合の重要な不変量はモチビック・コホモロジー，高次チャウ群とススリンホモロジーである．本研究ではこの不変量に対して，いくつかの定理が証明された．

研究成果の概要（英文）：Arithmetic geometry is the study of integral or rational solutions of systems of polynomial equations. For this, it is often useful to study the solutions in other domains, like complex number, real numbers, finite fields, or p-adic fields. An important invariant of such solution sets are motivic cohomology, higher Chow groups, and Suslin homology. During this project, I studied these invariants, and proved several interesting results about them.

研究分野：数論幾何学

キーワード：モチビック・コホモロジー　ススリンホモロジー　スキームの類対論

1. 研究開始当初の背景

The relationship of motivic cohomology, higher Chow groups and Suslin homology to other parts of arithmetic geometry has been established for a long time.

For an smooth and projective scheme over an algebraically closed field, Rojtman, Bloch and Milne proved that the albanese map induces an isomorphism of the torsion groups of $CH_0(X)$ and of the albanese variety of X. This result has been generalized to smooth (not necessarily projective) schemes by Spiess-Szamuely if one replaces the Chow group by Suslin homology and the albanese abelian variety by the albanese semi-abelian variety.

Kato-Saito proved that $CH_0(X)$ is finitely generated, and there is an injection with dense image $CH_0(X) \to \pi_1^{ab}(X)$ to the abelianized etale fundamtental group if X is smooth and proper over a finite field or regular, proper and flat over the $\mathbb{Z}$. This generalizes the finiteness of the class group and the isomorphism of the class group with the Galois group of the Hilbert class field in case $X = \operatorname{Spec} \mathcal{O}_K$. This result on class field theory has also been generalized to smooth, but not necessarily proper, schemes by Schmidt and Spiess.

The results above boil down to duality theorems and comparison between motivic cohomology and etale cohomology. Kato defined homology groups $H_i^K(X, \mathbb{Z}/m)$ measuring the difference between motivic cohomology and etale cohomology, and and conjectured that for regular and proper X over a number ring or finite field, $H_i^K(X, \mathbb{Z}/m)$ vanishes for $i \neq 0$. This is an important tool to relate motivic and etale cohomology and generalizes the theorem on class field theory mentioned above. Kato's conjecture was proven by Jannsen, S.Saito and Kerz (under resolution of singularities for the p-part in characteristic p). An analog of this conjecture was proven by S.Saito and Sato for regular proper varieties over complete discrete valuation rings.

2. 研究の目的

One of the main goals of this research was to generalize results on higher Chow groups to schemes over local and global fields. In particular, to establish the relationship between motivic cohomology and etale cohomology, and to see to which extend duality theorems hold is such situations. Moreover, I wanted to generalize the above results on class field theory and Rojtman's theorem to schemes which are not-necessarily smooth.

3. 研究の方法

Most of mathematical research is done with pencil and paper. But in order to exchange ideas, it is often necessary to communicate with other experts in the field. Many ideas evolve in informal discussions with other mathematicians.

(1) Organization of international conferences

In order to be up to date and learn about the newest development, I used the grant to help organize a yearly workshop on motives in Tokyo (together with S.Saito and T.Terasoma). I am also a coorganizer with A.Huber, M.Levine and U.Jannsen of the tri-annual conference on algebraic K-theory and motivic cohomology in Oberwolfach.

(2) Attendance of international and national conferences

I attended many conferences, both internationally and in Japan, where I presented my work, learned from other talks,

and discussed mathematics with other researchers. See below for the conferences I attended and gave presentations at.

(3) Joint reserach projects with foreign institutions

In order to generalize the results on class field theory of schemes to singular schemes, I worked together with A.Schmidt in Heidelberg. In order to facilitate this work, I visited Heidelberg once a year, and Schmidt came to work with me to Japan.

(4) International exchange

I visited several researchers at their institutes to discuss my research with them, and I invited researchers to discussion and give presentations to Japan.

4. 研究成果

(1) 主な成果

① The comparison of motivic cohomology and etale cohomology over henselian discrete valuation rings, was more difficult than anticipated, and is still in progress. As a start, I proved structure results and duality results about integral etale motivic cohomology over algebraically closed fields, finite fields, and local fields. This has been written up in a preprint and submitted for publication.

② I generalized Rojtman's theorem, stating that the torsion subgroup of the Chow group of zero cycles of a smooth projective variety over an algebraically closed field is isomorphic to the torsion subgroup of the Albanese semi-abelian variety. I showed that if one replaces the Chow group by Suslin homology then this holds for normal schemes (this was previously known for smooth schemes by work of Spieß-Szamuely) [5]. In order to achieve this, I first had to prove a descent theorem for Suslin homology [7]. I also wrote a survery article explaining the background of this theorem [6].

③ In an international collaboration with Alexander Schmidt (Universität Heidelberg, Germany) we studies class field theory of varieties over algebraically closed fields and finite fields [1], [2]. Over algebraically closed fields we showed that an explicit map constructed by us induces an isomorphism between the tame abelianized fundamental group mod m and the first Suslin homology group mod m (if either m is invertible in the field or if resolutions of singularities exist). Over finite fields, we proved that the enlarged tame abelian fundamental group is finitely generated (up to the part coming from the base-field). We then constructed a surjection of a modified version of Suslin homology (called Weil-Suslin homology) to this enlarged abelianized fundamental group, and proved that the kernel is the maximal divisible subgroup if resolution of singularities exist. In particular, we obtain an isomorphism of pro-finite completions. This generalizes work of Kato-Saito (for smooth and proper schemes) and Schmidt-Szamuely (for smooth schemes).

④ I proved two results which were not directly related to the project, but which I found while working on the project. The first is a duality for $\mathbb{Z}$-construcible sheaves for curves over finite fields [8], a generalization of a result of Lichtenbaum for smooth and proper curves to arbitrary curves. The second is a result giving several isomorphic versions of Parshin's conjecture, which states that higher algebraic K-theory of smooth and proper schemes over finite fields is torsion [4].

(2) 成果の国内外での位置付けと impact

My results are used and quoted by a other

mathematicians for their research. I was also invited to many international and national conferences to present my work, and invited to several universities to have seminar talks and discussion of my results.

(3) 今後の展望

I am continuing to work on motivic cohomology over global fields and local fields. My current project is to understand the motivic Tate-Shafarevich group, which expresses those motivic cohomology classes of varieties over a global field which vanish over each localization. I am trying to understand in which degrees this group can be expected to be finite.

5. 主な発表論文等

（研究代表者、研究分担者及び連携研究者には下線）

[雑誌論文](計 8 件)

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6. 研究組織

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