We developed new practical, finite-length lattice coding schemes to improve the spectral efficiency of wireless communication systems. We developed a new lattice encoding scheme that possesses three important properties: (1) satisfies the AWGN transit power constraint (high shaping gain) (2) has high error-correction capability (high coding gain) (3) efficient quantization and decoding algorithms (low computational complexity). We developed two new belief-propagation lattice decoding algorithms to improve the performance-complexity tradeoff. Lattice compute-and-forward techniques were applied to two Gaussian networks, showing that lattice coding can improve the spectral efficiency of wireless networks.
Wireless communications has become a fundamental societal necessity. From today’s smartphones, to tomorrow’s autonomous vehicles and the huge variety of “internet of things,” an increasingly large number of devices need to share a limited wireless spectrum. In addition, since many devices are battery operated, this must be done in a low power and computationally efficient manner.

Wirelessly networked devices communicate more efficiently by working together. They cooperate in their use of electromagnetic spectrum, where signals use real algebra. Lattices are error-correcting codes which provide reliable transmission using real-valued algebra.

The research goal is the development of practical lattices for reliable communication, and their application to wireless networks of networked wireless devices, hence the project name LatticeNet. This work concentrates on lattices, which are codes defined on the real numbers, which differs from error-correcting codes based on finite fields.

This work concentrates on the design of finite-length lattices, which differs from recent work on asymptotically long lattices. This work aims to exploit and investigate practical approaches to compute-and-forward, a recent theoretical technique to improve spectrum efficiency in Gaussian wireless networks.

The LatticeNet project is separated into five work packages WP1–WP5, the results are described for each one.

(1) WP1 Cubic Lattice Codes
For cubic lattice codes, two major results dealing with efficient decoding algorithms have been published:

① For decoding low-density lattice codes, we gave a Gaussian belief-propagation decoding algorithm which has the best performance-complexity tradeoff among all decoders, at low and medium dimension [論文4] [発表7].

For decoding low-density parity-check codes, used as a component of Code Formula lattices, our “max-LUT” method can result in performance better than belief-propagation decoding, while using only 4 bits/message, suitable for an efficient hardware implementation [論文6] [発表9].

These improved decoding algorithms are important both for rapidly evaluating lattice designs, and for power-efficient implementations in battery-powered devices.

In addition, a new construction of low-density lattice codes (LDLC) lattices based on the idea of array codes, borrowed from array low-density parity-check (LDPC) codes, was developed. A triangular matrix structure is highly suitable for practical encoding. These constructions outperform existing LDLC lattice constructions [発表12].

(2) WP2: Nested Lattice Codes & Quantization
We discovered and solved an important lattice coding problem. It was known that “nested lattice codes” can be constructed using separate coding lattices and shaping lattices. This is important because good coding lattices do not usually have efficient quantization algorithms. The problem we solved is how to encode information to lattice code points when the coding lattice and shaping lattices are distinct. This is an important practical aspect, since any communication system must encode information to lattice points. At the same time, the shaping lattice must be selected to have an efficient decoding algorithm. Figure 1 illustrates such a two-dimensional lattice code, and the corresponding encoding of information; the method is valid for lattices of any dimension.

Figure 1: Nested lattice code, where the shaping lattice (green) differs from the coding lattice (blue). While nested lattice codes are known, this research showed how to map information (blue numbers) to lattice code points. This is an important advance towards the use of lattice codes in practical systems.

The research method consists of the design of lattices and lattice codes, mathematical statements and their proofs, development of decoding algorithms and their software implementation. By separating into the these parts, we can deal with the problems systematically.
There are two types of lattices which are suitable for nested lattice codes, because there exist efficient quantization algorithms. A well-known lattice such as E8 or Barnes-Wall can be used as the shaping lattice and can be combined with a coding lattice to obtain 0.65 to 0.86 dB of shaping gain (out of a maximum of 1.53 dB) [1]. When the shaping lattice is based on a convolutional code, we showed a nested lattice code with 1.24 dB of shaping gain, when the coding lattice is a low-density lattice code [2]. This work has been generalized to shaping and coding lattices which satisfy specific conditions, and is currently under review for journal publication.

(3) WP3: Simple networks: Gaussian Relay Channels
The relay channel is a simple Gaussian network. Write-once memory (WOM) codes allow writing multiple times to a “write-once” memory. We showed that these codes are effective when applied to the relay channel. This is surprising since WOM codes were developed for data storage applications. We showed that WOM codes can achieve the maximum sum-rate for the asymmetric multiple access channel, for one specific rate pair. While WOM codes are not optimal in general, they induce an efficient decoding strategy, to reduce decoding complexity at the destination, when the source and relay transmit simultaneously [3]. This work has been generalized to shaping and coding lattices which satisfy specific conditions, and is currently under review for journal publication.

A more sophisticated Gaussian network has two sources, two relays and two destinations; part of this network is pictured in Figure 4. Lattice compute-and-forward can also be applied to this network. In this case, we applied random linear network coding (RLNC) to the network, and found that this generally improves performance in the form of reduced latency [4].

These results demonstrate that the theoretical promises of the compute-and-forward techniques can be realized by finite-length lattices to significantly improve spectrum utilization in wireless networks.
(5) WP5: Promotion of International Collaboration

Two mini-workshops were held, where project members gave progress report presentations. Overseas project members came to Japan with support from the Kakenhi budget. They participated in the workshops, provided feedback and suggestions for future direction. Overseas project members also gave invited seminars on their own research.

- August 25, 2014; held at University of Electro-Communications, Chofu, Tokyo.
- June 22–23, 2015; held at Japan Advanced Institute of Science and Technology, Nomi, Ishikawa.

As a result of this work package, research connections between researchers in Japan and overseas researchers have been strengthened, and are expected to lead to further collaborations, even after this project finishes.

5. Main Published Papers

- Conference Papers (7)


[Conference Papers (2)]


5. Y. Fujino, T. Wadayama, “A Construction of Non-Binary WOM Codes based on Integer...


