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研究成果の概要（和文）：私たちの提案は、超対称性や積分性などの特別な性質を仮定する必要なしに、量子フィールド理論（QFT）の特性とそれらの観測値に対する制約を研究するための新技術の開発に集中しました。このプロジェクトの成果は、大域的対称性を有する非ゼロ量子数を運ぶ状態および演算子に関連する観測値を系統的に近似する全く新しい方法を発明し、実質的に発展させることでした。既知の結果との接触や他の方法によるアクセスを可能にするために、私たちは主に、物理のさまざまなサブフィールドの他のグループによる可能な最大の理論的興味のあるクラスの理論に力を注いできました。

研究成果の概要（英文）：Our proposal was focused on the development of new techniques to study properties of quantum field theories (QFT) and constraints on their observables, which could be applied to generic quantum field theories, without the necessity of assuming any special properties such as supersymmetry or integrability. The achievement of this project was to invent, and substantially develop, an entirely new method for systematically approximating observables associate with states and operators carrying nonzero quantum number under a global symmetry. In order to make contact with known results or make possible access by other methods, we have mostly focused our efforts on classes of theories of the greatest possible theoretical interest by other groups in various subfields of physics.

研究分野： 理論物理学

キーワード： CFT　QFT　strong couling　supersymmetry　localization

１．研究開始当初の背景

To give the background for the large quantum number method is both difficult and easy. Easy, because the method was largely invented de novo in our preprint [1], described below. There was very little precedent, in any recent decades, for an analysis of quantum theories that has combined this level of precision with the broadness of scope and unviversal applicability we have achieved. As a result, the background on which our results were based, was quite sparse and can be described briefly. The difficulty is mainly in explaining why such a powerful and general method has gone unexploited until our work during the period of this project.

I will attempt here to provide some background and context in this section, both by describing some earlier work which our method was able to deepen and (vastly) expand, as well as explaining some of the global mental blocks and misunderstandings which prevented this method from being developed before the start of this project in 2015.

Regge theory and the effective string

The earliest known realization of a large quantum number expansion to study an otherwise unknown strongly-coupled theory, comes from Regge theory in the 1960s, via the observation by Chew and Frautschi that Regge trajectories $J(M^2)$ appear to be asymptotically linear in the large-J limit. This simplification at large spin led to the development of a classical string model to describe it, which birthed modern string thoery, where ultraviolet-complete string models were constructed by adding degrees of freedom in the form of additional compact dimensions and superdimensions, in order to preserve exact linearity of the Regge trajectory. It was only much more recently, through the development of effective string theory and the parallel development of the understanding of warped compactifications, that the linearity of the Regge behavior at large spin, was understood as an asymptotic rather than exact property at large J in planar QCD.

Together with Ian Swanson, the JSPS Kakenhi grantee and author of the present report (SH) developed the first systematic expansion of the mass of a string state as an asymptotic expansion at large spin, for mesons in the planar limit of QCD with bosonic quarks. This research resulted in striking discoveries, including a universal critical exponent, $J^{1/4}$, for the subleading correction to the mass-squared of the string state, together with a subleading correction scaling as J^0 whose coefficient, as well as exponent, is universal. These predictions follow directly from the structure of the effective field theory on the string worldsheet, with no need for any inputs from the deeper ideas of string theory, QCD, or any other complex inputs.

Conformal field theory at large spin

At the same time as large-quantum number methods were being applied to spinning strings, various groups were applying conformal bootstrap methods to study the spectrum of highly spinning primary operators at large spin. While the methods were vastly different from those used to study massive spinning sting states, the results had a strikingly familiar outcome: A tractable asymptotic expansion in negative powers of the spin, in which complicated strong-coupling interactions are collapsed into a small number of coefficients in the expansion, to any given order of approximation in the inverse spin.

Other CFT at large charge

Various isolated cases of operator dimensions at large spin and internal charge had been investigated in the decade prior to 2015, including the dimensions of monopole operators by Sachdev and collaborators, the famous BMN limit by Maldacena et al, and work on anomalous dimensions in the large spin limit in near-conformal QCD by Basso et al.

２．研究の目的

The purpose of the research, as we've emphasized above, is to move beyond isolated cases that have already been examined, and to ask the broader question: Why do strongly coupled theories simplify at large quantum number, in general? What is the explanation for this pattern? What general methods may be used to derive the large quantum-number expansion for different quantum numbers, for different observables, in different types of theories? What other types of theories, observables, and quantum numbers, beyond those already studied, may usefully be studied with such an expansion?

The tools currently in general use to study interacting quantum field theories prior to our project, have been sharply limited. To a great extent, the main tools used are:

Perturbation theory, numerical methods (mostly Monte Carlo), and supersymmetric models of strong coupling phenomena. More recently strong/weak coupling dualities and holographic dualities have emerged as potentially tractable tools for analyzing practical questions in quantum field theory.

While useful, some of these previously-existing tools are sharply limited in scope: Perturbation theory in the usual sense can be used only when the Hamiltonian of the theory is represented as a weak perturbation of a noninteracting system. Supersymmetric methods are applicable only to supersymmetric models, and the behaviors of those models may or may not extrapolate to more general systems. Dualities are applicable only when the dual Hamiltonian is known and despite steady progress in working out examples, no systematic understanding exists of duality, let alone a general recipe for constructing duals, either strong/weak or holographic. Monte Carlo simulations have produced limited results outside of a few special cases for which algorithms have been specially optimized and lattice artifacts carefully tuned away by renormalization.

Much more recently, the conformal bootstrap has emerged as a major method for learning about the behavior of strongly interacting conformal field theories. The modern renaissance of bootstrap methods has solved the spectrum and interactions of the 3D Ising model to an accuracy now surpassing other existing methods. Despite this success, the numerical bootstrap has its own apparent limitations, slowing down exponentially with external operator dimension due to the exponential growh of the dimension of the linear programming problem involved. Relatedly, bootstrap-type methods and conformal truncation run into similar computational barriers for nonconformally perturbed CFT in the infrared for the same reasons. The success of the bootstrap analysis of the 3D Ising model has not been replicated for other theories, even simple ones, for quite ineluctable reasons.

It is therefore desirable to find methods that allow attack of generic problems in a computationally robust way, without imposing special conditions (other than a global symmetry of some kind) on the choice of theory. The large quantum number expansion we have developed in this project, was developed to accomplish this goal, and it has been successful now in many strongly coupled systems.

３．研究の方法

Effective field theory in the large-charge expansion

The most generally applicable method used to analyze strongly coupled systems has been effective field theory (EFT) in the sense advocated by Kenneth Wilson: One describes the QFT as a set of degrees of freedom with a momentum cutoff, whose Hamiltonian or Lagrangian is an infinite sum of terms with arbitrary number of derivatives.

In generic theories, this description is of limited usefulness, because the terms depend on the form and magnitude of the cutoff. This is certainly the case for the many models we have studied, which are described as field theories whose interaction Hamiltonian matrix elements are of order 1 for any basis of elementary particle excitations.

In the large charge regime, however, remarkable simplifications arise in the Wilsonian description. Though the effective Lagrangian is a property of the theory rather than the state, the system simplifies dramatically when the scale of the charge density is taken parametrically higher than the Wilsonian cutoff. In this regime, the effective action re-organizes itself in an expansion where the cutoff appears in the numerator only, and the charge density appears to arbitrarily high powers in the denominator to compensate the dimensionality of the cutoff and fields and their dervatives in the numerator.

The result is that both higher-derivatives on the one hand, and loop corrections on the other hand, are suppressed when we take a large hierarchy between the infrared scale and the charge density, with the Wilsonian cutoff taken to lie between the two.

If we further restrict to the case of a conformal field theory, then the renormalization group (RG) equation says observables are independent of the cutoff, which further restricts the properties of the Lagrangian: The leading terms in the expansion in the cutoff must be *classically* conformally invariant and cutoff-independent, and the cutoff-dependent terms are determined algorithmically by the RG fixed point equation from the leading, cutoff-dependent terms order by order in the cutoff.

Furthermore, at large density many degrees of freedom which are present in the full effective lagrangian at the conformal fixed point, may get large frequencies and be integrated out of the system for purposes of large-density computations. The result is that the system at large density can generally be described by an action which is far simpler than the underlying CFT, in the sense that it contains fewer degrees of freedom, and is strongly coupled, even if the full CFT is strongly coupled, and has many degrees of freedom.

For a given set of light fields and symmetries, the renormalization group equation does *not* fix the leading (= classical) terms in the large-charge EFT. However the organization of the large-charge EFT implies that terms with more curvatures, fields, and derivatives, are suppressed by increasing numbers of powers of the density. The result is that any observable can be calculated to any given order in the ratio of the infrared scale to the density scale, given a finite number of leading coefficients in the large-charge EFT.

Operator dimensions from radial quantization

Many interesting observables can be studied using the large-charge EFT, at scales far below the charge density. But the simplest sharply-defined data of a CFT are the scaling dimensions of primary local operators. In weakly-coupled systems these are close to integer or half-integer, but in strongly coupled systems they may take any real value, outside of regions excluded by unitarity bounds.

By virtue of the "state-operator correspondence", a rule applicable to all unitary CFT, the set of scaling dimensions of local operators of a theory is equal to the spectrum of the Hamiltonian acting on the Hilbert space of a system with spherical spatial slice of unit radius.

For local operators of large global charge, then, the state on the sphere is automatically in the regime where the large-charge EFT is applicable, for observables such as the ground state energy, where the infrared scale is the size of the sphere itself. It follows that the dimension of the lowest lying primary state with charge J (and other low-lying excited states) has a dimension that is calculable in a series in 1/J, using the large-charge EFT.

Application to the Wilson-Fisher O(2) model

The first target of our method was the critical (conformal) point of the Wilson-Fisher O(2) model in three spacetime dimensions, where the large quantum number J is just taken to be the charge under the continuous Abelian subgroup of the O(2) symmetry. The EFT at large charge is remarkably simple: The content consists of one (periodically identified) scalar field χ only, which can be thought of as the phase variable of the complex scalar in the usual weakly coupled linear sigma model flowing to the conformal fixed point.

The leading large-charge Lagrangian density is the simplest possible conformal effective Lagrangian one can write for such a field, preserving the shift symmetry of the phase χ.

This phase degree of freedom can be thought of in some sense as a "goldstone boson" of the internal SO(2) symmetry. The sense in which this degree of freedom is a goldstone field was later explored and made more precise in follow up work by Alvarez-Gaume et al, and by Rattazzi et al, who showed that the large-charge EFT of the O(2) model can be understood as realizing a new pattern of spontaneous breaking of combined spacetime and conformal symmetries. This pattern can be fit into the conceptual framework of the Coleman-Callan-Wess-Zumino (CCWZ) analysis of spontaneous symmetry breaking, but nonetheless had not been considered previously in that approach.

The computation of operator dimensions in the large-charge EFT yields strikingly sharp constraints on the form and coefficients of the operator dimensions at large charge. The lowest dimension operator of charge J is found to be a scalar with dimension

$\Delta_J=c_{3/2} J^{3/2} + c_{1/2} J^{1/2} - 0.0937256\ldots + 0 (J^{-1/2})$

The two leading coefficients are in principle fixed but not precisely known numbers corresponding to coefficients in the EFT lagrangian. The absence of a $J^{1/2}$ term and the universal value of the constant term, follow from the field content and structure of the EFT, independent of the symmetries.

Quantization of the EFT leads to many further relations and constraints on the spectrum at large J. For instance, the second lowest primary at large J has spin two and its dimension lies above the scalar ground state by an amount equal to $\sqrt{3}$, plus terms vanishing at large J.

Large-charge universality classes

The simplicity of the large-charge EFT suggests many of the details of the underlying CFT are washed out, and indeed this is so: Many distinct CFT may correspond to the same large-charge universality class, with different values of the coefficients $c_{3/2}$, $c_{1/2}$, but the same field content, structure of the EFT, and the universal properties such as the J^0 term 0.0937256. Thus, by analyzing one single large-charge EFT we may derive constraints on large-charge observables for many different CFT at once.

For instance, the N=2 superconformal fixed point with a single chiral superfield and cubic superpotential, lies in the same universality class at large R-charge, with the fermions becoming heavy on the density scale (via a supersymmetric version of the Brauner-Murayama-Watanabe massive-goldstone mass formula) and decoupling. At large R-charge, the only light degree of freedom is the phase in the chiral superfield, the EFT is the same as in the O(2) model, and the structure of the asymptotic expansion (including the value of the universal term) is the same as in the O(2) model, but with different coefficients.

Similarly, the conformal fixed points of the three-dimensional CP_n sigma models, can be studied at large soliton charge J, or equivalently large mangnetic monopole charge in the linear sigma model realizaton as multiflavor scalar electrodynamics. The large-monopole-number limit describes the same large-charge universality class as the O(2) model, and indeed at large charge we have constructed explicitly the duality map between the O(2) model phase variable and the magnetic flux.

In very recent work by A. de la Fuente, the large-monopole-number expansion has been worked out explicitly for the CP_n model to leading and subleading order in n. The results confirm both the structure of the asymptotic expansion and the value of the J^0 term.

Other large-charge universality classes

Other simple CFT in 3 and 4 dimensions realize different large-charge universality classes. We have examined several different examples using the same methods. Examples analyzed have included:

・The conformal SO(2n) sigma models at large quantum numbers under The nonabelian symmetries, particularly theSO(4) model with two Commuting charges taken large in fixed ratio;

・The simplest N=2, D=3 supersymmetric Wess-Zumino model with a one dimensional vacuum manifold and non-R global symmetry;

・ The general case of N=2, D=4 superconformal field theories with One dimensional Coulomb branch, at large U(1) R-charge.

In all these cases we use the appropriate effective field theory to compute operator dimensons and OPE coefficients, as an asymptotic expansion at large charge. In the latter two cases, we are able to compare with exact supersymmetric methods and find precise nontrivial agreement everywhere our results can be checked.

４．研究成果

Major outcome of the research

The main outcome of our project is the development of a new technique that allows the analysis of operator dimensions and operator product coefficients in states of large global charge, in fully generic quantum field theories with global symmetries. The predictions of our method have been confirmed by every method with which they have been checked, including conformal bootstrap, Monte Carlo simulations, large-n expansions, and supersymmetric localization and other suprsymmetric methods in the case where the charge is an R symmetry quantum number.

Its positioning and impact in domestic and overseas trends

The positioning in overseas trends is obviously very good: Several groups overseas have already begun following and extending our methods, particularly groups in Europe (CERN, Lausanne, Bern, Torino) and the United States (Harvard, Simons Center at Stony Brook).

Future prospects

The future prospects are very promising to extend our method to many other cases, and solve everything.

５．主な発表論文等

（研究代表者、研究分担者及び連携研究者には下線）

〔雑誌論文〕（計　４件）

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〔学会発表〕（計１２件）

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６．研究組織

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