様　式　Ｃ－１９、Ｆ－１９－１、Ｚ－１９　（共通）

科学研究費助成事業　　研究成果報告書

平成　２９　年　　６　月　１２　日現在

機関番号：　１３９０１
研究種目：　若手研究(A)
研究期間：　2014～2016
課題番号：　２６７０７００５

研究課題名（和文）Probing crystal defects with scattering theory and non-commutative topology

研究課題名（英文）Probing crystal defects with scattering theory and non-commutative topology

研究代表者

Ｒｉｃｈａｒｄ　Ｓｅｒｇｅ（Richard, Serge）

名古屋大学・多元数理科学研究科（国際）・G30特任教授

研究者番号：７０７２５２４１

交付決定額（研究期間全体）：（直接経費）　　5,000,000円

研究成果の概要（和文）：種々の電場または磁場を伴うSchroedinger作用素のスペクトルおよび散乱の結果が得られた。我々の研究は、必然的に散乱理論における指数定理につながるものである。散乱理論における指数定理は、摂動の下で安定している量の出現に対応する。また我々は、異方的な1次元量子ウォークのスペクトル解析を十分に発展させ、任意個の連続スペクトルに埋め込まれた束縛状態をもつSchroedinger作用素の構成法を得た。

研究成果の概要（英文）：Spectral and scattering results for Schroedinger operators with various type of electric or magnetic potentials have been obtained. Our investigations naturally lead to index theorems in scattering theory. Such results correspond to the exhibition of quantities which are stable under perturbations. We have also fully developed the spectral analysis of 1 dimensional quantum walk with anisotropic behavior, and provided a construction for embedding an arbitrary number of eigenvalues in the continuous spectrum of a Schroedinger operator.

研究分野：　mathematics

キーワード：　functional analysis　spectral theory　scattering theory　index theorem

1) 研究開始当初の背景 Background of the research

Prior to this project we had put into evidence a C*-algebraic framework which leads naturally to index theorems in scattering theory. The general theory had been applied to a few models in the context of potential scattering. With this approach we were able to show that a theorem firstly established in 1949 (Levinson's theorem) and then extensively studied by several researchers was in fact an index theorem. With such an approach, the stability of the result under perturbations or deformations is automatically established.

2) 研究の目的 Purpose of research

The purpose of this research was to further extend this topological approach of Levinson's theorem, especially in the context of crystals with defects. Indeed, the algebraic framework is flexible enough for expecting a generalization of such index theorems to a much wider class of models. The fundamental tools of this approach are, an abstract K-theoretical index theorem, and Connes-pairing between K-group elements and cyclic cocycles. As a result one obtains an equation between topological invariants, or in other words an equality between quantities which are stable under perturbations. In this project we expected to relate topological properties of crystals to topological properties of the scattering matrix. We also wanted to develop further models and extend the applicability of our framework.

3) 研究の方法 Methods of research

Since this research was mainly based on the study of models of quantum systems we spent a lot of time performing spectral and scattering theory on such models. These investigations were realized in collaboration with several researchers worldwide, and together with my PhD student D. Parra. Thus, during the three years of this proposal we visited some collaborators in Japan, in France, in Chile, and in Australia and we also invited some of them for collaboration in Japan. Altogether we intensively collaborate with 6 researchers during this period. We also attended several conferences and delivered numerous talks for discussing the results of our investigations.

4) 研究成果 Research results

As indicated below, the outcomes of this period of research consists in 8 published papers and in 4 additional papers submitted for publication. Some of these investigations are directly linked with the initial project, some are focusing on other aspects of spectral and scattering theory which have been encountered during our investigations.

In publications [2,3,11] we concentrate on periodic structures, in the continuous setting in [2] and in the discrete setting in [3,11]. In [2] we develop the spectral and scattering theory for a simplified model of a crystal structure in a half-space. In [3] the spectral theory of very general magnetic operators on Z^d is considered and some continuity properties of the spectrum are exhibited. The main difficulty for such investigations is to get rid of the magnetic potential and to concentrate on the magnetic field only. A C*-algebraic framework involving twisted crossed product has been used for that purpose. In [11] perturbations of periodic operators are considered and general spectral and scattering results are presented. These investigations take place in the very general framework of crystal lattices, and are necessary steps for our long-term project.

In the papers [1,7,8,10] the C*-algebraic framework is either further extended or used for obtaining index theorems in the context of scattering theory. The review paper [7] contains the main ideas about our investigations on a topological approach of Levinson's theorem. Paper [8] contains some preliminary investigations for Dirac operators, while publications [1] contains an exhaustive study of a model of Schroedinger operators with a very singular potential at the origin. Based on this model, we have been able to further extend the applicability of our C*-algebraic framework. As a consequence, we obtain in [10] for the first time a Levinson's type theorem for system with complex eigenvalues (and not only real eigenvalues) and we are now considering

the case of an infinite number of eigenvalues. Since this situation often takes place for perturbed crystals, such investigations are also a necessary step for our long-term project on crystals.

The publications [5] and [6] are concerned with a thorough analysis of phenomena taking place at thresholds. Indeed, at these special spectral values usual arguments involving a limiting absorption principle usually break down. Since the C*-algebraic framework requires a good understanding of phenomena taking place at these values, we have studied them separately.

In [4] we show how n positive eigenvalues can be embedded into the continuous spectrum of a Schroedinger operator in 1D. Such investigations are an extension of the classical work of Wigner and von Neumann. In [9] the spectral analysis of a one dimensional quantum walk with 2 different asymptotic operators is performed. Part of this analysis is based on an earlier work on Mourre theory in a two Hilbert spaces setting. However, the framework had to be adapted to unitary operators, since it existed only for self-adjoint operators. Finally, the short paper [12] is a generalization of earlier results in the context of dynamical systems. More precisely, we present a new criterion, based on commutator methods, for the strong mixing property of unitary representations of topological groups equipped with a proper length function.

As a conclusion, one can say that the final goal of the original proposal has not been reached yet. However, we have considered several preliminary steps and our understanding of the problem has increased a lot. On the way towards this ultimate goal, several parallel investigations have been performed and the global result of these three years is very positive. In the future, we plan to go on with this proposal and with the investigations on perturbed crystals.

5) 主な発表論文等 Research results

Publications:

Total 12 (8 published paper + 4 submitted)

1) J. Derezinski, S. Richard, *On Schrödinger operators with inverse square potentials on the half-line*, Ann. Henri Poincaré **18** (2017), 869 – 928. DOI 10.1007/s00023-016-0520-7

2) S. Richard, R. Tiedra de Aldecoa, *Spectral and scattering properties at thresholds for the Laplacian in a half-space*, J. Math. Anal. Appl. **446**, 1695 – 1722, 2017. DOI http://dx.doi.org/10.1016/j.jmaa.2016.09.045

3) D. Parra, S. Richard, *Continuity of the spectra for families of magnetic operators on Z^d*, Anal. Math. Phys. **6**, 327 – 343 2016. DOI 10.1007/s13324-015-0121-5

4) S. Richard, J. Uchiyama, T. Umeda, *Schrödinger operators with n positive eigenvalues: an explicit construction involving complex valued potentials*, Proc. Japan Acad. **92**, Ser. A, 7 – 12, 2016. DOI 10.3792/pjaa.92.7

5) S. Richard, T. Umeda, *Low energy spectral and scattering theory for relativistic Schrödinger operators*, Hokkaido Mathematical Journal **45**, 141 – 179, 2016.

6) S. Richard, R. Tiedra de Aldecoa, *Resolvent expansions and continuity of the scattering matrix at embedded thresholds: the case of quantum waveguides*, Bull. Soc. math. France **144**, 251 – 277, 2016.

7) S. Richard, *Levinson's theorem: an index theorem in scattering theory*, Proceedings of the Conference *Spectral Theory and Mathematical Physics*, Santiago 2014, Operator Theory Advances and Applications **254**, 149 - 203, Birkhäuser, 2016.

8) K. Pankrashkin, S. Richard, *One-dimensional Dirac operators with zero-range interactions: Spectral,*

scattering, and topological results, J. Math. Phys. **55**, 062305-1 – 062305-17, 2014. DOI http://dx.doi.org/10.1063/1.4884417

Submitted publications

9) S. Richard, A. Suzuki, R. Tiedra de Aldecoa, *Quantum walks with an anisotropic coin I: spectral theory, submitted.*

10) F. Nicoleau, D. Parra, S. Richard, *Does Levinson's theorem count complex eigenvalues?*, submitted.

11) D. Parra, S. Richard, *Spectral and scattering theory for Schrödinger operators on perturbed topological crystals*, submitted.

12) S. Richard, R. Tiedra de Aldecoa, *Commutator criteria for strong mixing II: More general and simpler*, submitted.

Delivered presentations:

Total 13 (2 presentation at conferences, 1 presentation at workshop, 1 intensive course, 9 seminar presentations)

1) Seminar of mathematical physics, WPI AIMR Tohoku University, Sendai (JP), February 23, 2017. Title: *Spectral and scattering theory on perturbed crystal lattices.*

2) Seminar of mathematical physics, Université de Paris-Sud / Orsay (F), September 21, 2016. Title: *Opérateurs de Schroedinger presque homogènes.*

3) Seminar of mathematics physics, Université de Grenoble (F), September 19, 2016. Title: *Opérateurs de Schroedinger presque homogènes.*

4) Seminar of EDP, analysis and applications, Université de Lorraine / Metz (F), September 16, 2016. Title: *Opérateurs de Schroedinger presque homogènes.*

5) Seminar of analysis, geometry and algebra, Université de Lorraine / Metz (F), September 15, 2016. Title: *Continuité du spectre pour des familles d'opérateurs magnétiques sur Z^d.*

6) Seminar of analysis, Université de Bordeaux (F), September 12, 2016. Title: *Théorie spectrale et de la diffusion sur des cristaux topologiques perturbés.*

7) Intensive course, School of mathematics, UNSW, Sydney (Australia), March 7 - 11, 2016. Title: *Spectral theory, scattering theory, and index theorems.*

8) Conference **Spectral and Scattering Theory and Related Topics**, Kyoto (JP), January 20 - 22, 2016. Title: *Continuity of the spectral for families of magnetic operators on Z^d.*

9) Seminar of analysis, University of Hyogo, Himeji (JP), November 28, 2015. Title: *Continuity of the spectral for families of magnetic operators on Z^d.*

10) Workshop **Quantum walks, random walks and scattering theory on periodic systems**, Nagoya (JP), January 21 - 22, 2015. Title: *Towards scattering theory and index theorems in periodic systems.*

11) Seminar of analysis, University of Tsukuba (JP), December 10, 2014. Title: *Asymptotic expansion at embedded thresholds.*

12) Conference **Spectral Theory and Mathematical Physics**, Santiago (Chile), November 24 - 28, 2014. Title: *Levinson's theorem: an index theorem in scattering theory.*

13) Seminar of operator algebras, University of Tokyo (JP), October 1, 2014. Title: *Back- and-forth between scattering theory and index theorems*.

6) 研究組織

研究代表者
リシャール　セルジュ（Serge RICHARD）
名古屋大学　多元数理科学研究科（国際）

G30 特任教授

研究者番号: 70725241