# 科研費

# 科学研究費助成事業 研究成果報告書

平成 29 年 5 月 28 日現在

機関番号: 13901 研究種目: 若手研究(B) 研究期間: 2014~2016

課題番号: 26800008

研究課題名(和文)ポテンシャル付き箙によるクラスター代数の圏化

研究課題名(英文) Categorification of cluster algebras by quivers with potential

#### 研究代表者

DEMONET Laurent (DEMONET, Laurent)

名古屋大学・多元数理科学研究科(国際)・G30特任准教授

研究者番号:70646124

交付決定額(研究期間全体):(直接経費) 2,400,000円

研究成果の概要(和文):研究期間内における主な結果は以下である。整環を用いて団代数の圏化を研究した。特に、Luoとの2本の共著論文においては組み合わせ論的モデル(多角形の三角形分割)の圏化を調べ、伊山との共著論文においてはLie理論(旗多様体)における圏化を研究した。伊山、Jassoとの共同研究では、多元環上の傾加群とねじれ類を調べ、その幾何学的実現をKO群を用いて与えた。さらに、それらを有限個しか持たない多元環(傾有限多元環)の特徴付けを与えた。また単著論文において、部分三角形分割から定まる新しい多元環のクラスを導入した。これはBrauerグラフ多元環と、曲面のから生じる多元環を、共通に拡張するクラスである。

研究成果の概要(英文): The main results achieved during this project are the following. I studied orders in categorification of cluster algebras. In three articles (two with X. Luo and one with O. Iyama), we investigated one the one hand categorification of combinatorial models (triangulations of polygons) and on the other hand categorification coming from Lie theory (partial flag varieties). An other project (with O. Iyama and G. Jasso) consisted to study support tau tilting modules and torsion classes in the case of algebras having finitely many of them. In particular, we reached nice geometric realizations and good characterizations of those algebras. Finally, I introduced by myself a new class of algebras (algebras of partial triangulations) containing Brauer graph algebras as well as Jacobian algebras of surfaces. Moreover, this class of algebras have important properties (it has finite rank, has an easy to understand tilting theory and is of tame representation type).

研究分野: algebra

キーワード: representation theory finite dim algebras categorification cluster algebras

#### 1.研究開始当初の背景

This project have been conducted in representation theory of algebras and particularly in categorifications problems of cluster algebras. This relatively new field (it started around 2000) have been proved useful in several context (it permitted to prove the celebrated periodicity conjecture for example).

## 2.研究の目的

The main purpose of this research has been to understand several mutations systems in categories as well as their applications to categorification.

# 3.研究の方法

A part of this research has consisted in collaborations and another part has been by myself. As this is research in pure mathematics, the important part of collaboration consists in meeting other people, either by traveling or by inviting colleagues.

#### 4.研究成果

The main results achieved during this project are the following. I studied orders in categorification of cluster algebras. In three articles (two with X. Luo and one with O. Iyama), we investigated one the one hand of categorification combinatorial models (triangulations of polygons) and on the other hand categorification coming from Lie theory (partial flag varieties). An other project (with O. Iyama and G. Jasso) consisted to study support tau tilting modules and torsion classes in the case of algebras having finitely many of them. In particular, we reached nice geometric realizations

and good characterizations of those algebras. Finally, I introduced by myself a new class of algebras (algebras of partial triangulations) containing Brauer graph algebras as well as Jacobian algebras of surfaces. Moreover, this class of algebras have important properties (it has finite rank, has an easy to understand tilting theory and is of tame representation type).

# 5.主な発表論文等

(研究代表者、研究分担者及び連携研究者には下線)

### [雑誌論文](計 件)

1. <u>L. Demonet</u>, O. Iyama, Lifting preprojective algebras to orders and categorifying partial flag varieties, Algebra Number Theory 10, no. 7, 1527–1579 (2016).

DOI: 10.2140/ant.2016

2. <u>L. Demonet</u>, X. Luo, Ice quivers with potential arising from once-punctured polygons and Cohen-Macaulay modules, Publ. Res. Inst. Math. Sci. 52, no. 2, 141–205 (2016).

DOI: 10.4171/PRIMS/177

3. <u>L. Demonet</u>, X. Luo, Ice quivers with potential associated with triangulations and Cohen-Macaulay modules over orders, Trans. Amer. Math. Soc. 368, no. 6, 4257-4293 (2016).

DOI: 10.1090/tran/6463

4. <u>L. Demonet</u>, Introduction to algebras of partial triangulations, Proceedings of the 49th Symposium on Ring and Representation Theory. 5 pages (2016).

L. Demonet, P.-G. Plamondon, D. Rupel, S. Stella, P. Tumarkin. SL 2 -tilings do not exist in higher dimensions (mostly), accepted to Sém. Lotharingien de Combinatoire, 4 pages.

[学会発表](計 件)

- 1. <u>L. Demonet</u>, Orders categorifying cluster algebras structures of partial flag varieties", seminar in Bonn university, 2016/05/20.
- 2. <u>L. Demonet</u>, Algebras of partial triangulations, seminar in Roma university, 2016/03/30.
- 3. <u>L. Demonet</u>, Algebras of partial triangulations", seminar in Paris 7 university, 2016/03/25.
- 4. L. Demonet, Algebras of partial triangulations, seminar in Dijon university, 2016/03/24.
- 5. <u>L. Demonet</u>, Algebras of partial triangulations, Workshop on Brauer Graph Algebras in Stuttgart, 2016/03/22.
- 6. <u>L. Demonet</u>, Orders categorifying cluster algebras structures of partial flag varieties, Workshop on Cluster Algebras and Geometry in Münster university, 2016/03/10.
- 7. <u>L. Demonet</u>, Algebras of partial triangulations, seminar in Bielefeld university, 2016/03/09.
- 8. <u>L. Demonet</u>, Algebras of partial triangulations, seminar in Caen university, 2016/03/01.
- 9. <u>L. Demonet</u>, Categorification of Cluster Algebras, series of 3 lectures in Winter School about Lie algebras and related topics, Tokyo University, 2016/01/08, 2016/01/09 and

2016/01/10.

- 10. <u>L. Demonet</u>, Orders categorifying cluster algebras structures of partial flag varieties, Workshop on Homological Interactions between Representation Theory and Singularity Theory in Edinburgh, 2014/12/18.
- 11. <u>L. Demonet</u>, Cohen-Macaulay modules over orders associated with triangulations and cluster categories (type A and D), Cluster Algebras in Combinatorics and Topology in Seoul, 2014/12/16.
- 12. <u>L. Demonet</u>, Orders categorifying cluster algebras structures of partial flag varieties, Cluster Algebras and Representation Theory in Seoul, 2014/11/06.

[図書](計 件)

〔産業財産権〕

○ 出願状況(計 件)

名称:

発明者:

権利者:

種類:

番号:

出願年月日:

国内外の別:

○取得状況(計 件)

名称:

発明者:

権利者:

種類:

番号:

取得年月日:
国内外の別:
〔その他〕
ホームページ等
http://www.math.nagoya-u.ac.jp/~demonet/
6.研究組織
(1) 研究代表者 Laurent Demonet,
Nagoya University, Graduate School of
Mathematics.
研究者番号: 70646124
(2) 研究分担者
( )
研究者番号:
(3) 連携研究者
( )
研究者番号:
(4) 研究協力者
( )